A

TREATISE

ON THE

STEAM ENGINE,

Historical, Practical, and Descriptive.

[v. 1]

BY

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Voilà la plus merveilleuse de toutes les Machines; le Mécanisme ressemble à celui des animaux. La chaleur est le principe de son mouvement; il se fait dans ses différens tuyaux une circulation, comme celle du sang dans les veines, ayant des valvules qui s'ouvrent et se ferment à propos; elle se nourrit, s'évacue d'elle-même dans des tems réglés, et tire de son travail tout ce qu'il lui faut pour subsister. Cette Machine a pris sa naissance en Angleterre, et toutes les Machines à feu qu'on a construites ailleurs que dans la Grande Bretagne ont été éxécutées par des Anglois. BELIDOR, *Architecture Hydraulique.*

ILLUSTRATED BY

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CHAPTER VII.

Application of the sliding rule for calculating the dimensions for the parts of Steam-Engines.

Mr. Watt proportioned all the parts of his patent rotative-engines so judiciously, that after a few years' practice in making those engines, he ascertained the proper proportions for every part, and established standards for the dimensions of engines of all sizes; these dimensions have been followed ever since, by the best engineers and makers of steam-engines, with very few deviations, because long experience has proved that those standards were extremely well proportioned.

In this part of his subject, Mr. Watt was greatly assisted by several ingenious workmen and operative engineers, who had been educated under his own eye, in the manufactory at Soho, and who had acquired a stock of experience in the course of practice. The calculations which were required for proportioning the dimensions of engines, were commonly intrusted to Mr. Southern, who was a skilful mathematician, and to whom Messrs. Boulton and Watt were induced to give an interest in their manufactory chiefly on that account. Mr. Watt, with the assistance of Mr. Southern, investigated all the circumstances which can affect the proportions of each part of a steam engine; and thence formulæ were deduced by which the dimensions could be calculated for each individual case. The dimensions so ascertained were communicated to the workmen for their guidance, but the rules themselves, or the principles of calculation which were followed, are very little known.

At the same time Mr. Watt employed logarithmic scales, on a sliding rule, for performing calculations relative to steam-engines and machinery. These instruments had been long in use amongst gaugers and officers of the excise, and were also used by carpenters; but they were very coarsely and inaccurately divided, and required some improvements to render them serviceable to engineers. Mr. Watt and Mr. Southern arranged a series of logarithmic lines upon a sliding rule, in a very judicious form, and they employed the most skilful artists to graduate the original patterns, from which the sliding rules themselves were to be copied.

The Soho sliding rules are made of box-wood, $10\frac{1}{2}$ inches long, with one slider, and four logarithmic lines on the front face; and at the back are tables of useful numbers, divisors and factors, for a variety of calculations. Sliding rules of this kind are still called the Soho rules, and they are so correctly divided by some of the best makers of mathematical instruments in London, that they are capable of performing ordinary calculations with sufficient accuracy for practice; and by means of the tables at the back of the rule, most questions in mensuration may be very readily solved.

These sliding rules were put into the hands of all the foremen and superior workmen of the Soho manufactory, and through them, the advantage of calculating by means of the sliding rule has become known amongst other engineers, and some do employ it for all computations of ordinary mensuration; but the habit of using it upon all occasions, is almost confined to those who have been educated at Soho. To apply the sliding rule extensively for the calculation of the dimensions of the parts of steam-engines and machinery, particular formulæ are required, which were confined to a very few of the principal engineers at Soho, and have not been at all disseminated in the profession.

The great number of patent engines which Messrs. Boulton and Watt sent out, and fixed in every part of the kingdom, furnished a sufficient number of models, from which other engineers could easily ascertain the proper dimensions for the parts of an engine of any particular size which they might be required to construct. The knowledge of these dimensions has also been made generally known by workmen who were brought up at Soho, and who have by degrees become dispersed over the kingdom; but they have rarely acquired, or been able to communicate, much knowledge of the principles by which the dimensions are regulated (*a*).

From the great experience which engineers have acquired since Mr. Watt's time, it may be presumed, that if any considerable deficiencies or errors had existed in the dimensions of his standard engines, they would have been corrected in the modern engines. This has actually been the case, in some few instances; but in almost all essential particulars, the practice of the most skilful engineers of the present day, is very nearly the same as that of Mr. Watt himself; and in those few instances where they differ, the modern practice is for the most part inferior to the original, which ought to be studied with care by all engineers, as the fountain-head for that kind of knowledge. In that view the information contained in this and the next chapter will be very useful to the profession, and accordingly the author has taken great pains to verify all the proportions and rules which he has formed, by a continual reference to Mr. Watt's own practice, so as to be assured of their correctness.

The properties of the logarithmic lines upon a sliding rule are not very generally known to practical engineers, and there is no complete treatise extant upon the subject (*b*), hence it becomes necessary for the instruction of students, to supply

(*a*) The author, at his first entrance into business, made it his particular study to acquire a complete knowledge of the structure of Mr. Watt's steam-engines, and of the proportions and dimensions of all their parts; as being in every respect the very best course of instruction for a practical mechanician. With this view, in the years 1804 and 1805, he examined and took exact drawings of a number of those engines of all sizes, with their dimensions; and after having accumulated a sufficient collection of observations, they were arranged and compared, to find out the proportions that the different dimensions bear to each other; which being ascertained, corresponding rules were formed for calculating the dimensions, in every case, either by common arithmetic, or by the sliding rule.

The author is not aware, whether the rules which he thus made himself, are exactly the same as those which Messrs. Boulton and Watt followed; but the rules in question have been proved in the course of several years' practice, and corrected when necessary, so as to give results which upon an average, correspond very nearly with the practice of the most experienced engineers, who have all taken their proportions from the established models of Messrs. Boulton and Watt's standard engines.

(*b*) The author procured a Soho sliding rule at his first commencement in business; but not being able at that time to obtain any instructions for the mode of using it, and having observed the facility with which the Soho workmen performed their ordinary calculations by it, he was induced to investigate the properties of the instrument very fully, and thence deduced formulæ for its application upon all occasions.

The same course had been begun a little earlier by Mr. Benjamin Bevan, civil engineer and architect, who has since published a practical treatise on the sliding rule, in octavo, 1822; this is a valuable little work, which contains a collection of useful theorems for performing all kinds of calculations; but as it is intended for the instruction of all professions who require calculations, and not particularly for any one class, it does not contain many of those specific formulæ which render the instrument particularly valuable to mechanicians.

A treatise on the sliding rule was published some years ago, by Dr. Mackay, for the particular instruction of navigators, to enable them to perform nautical calculations by it. A small book has also been printed by Mr. Routledge, engineer, of Leeds, to explain the Soho sliding rule, and promote the use of it amongst engineers: it is sold by the makers of those rules.

that deficiency, and to explain the construction of the various formulæ which are given in this work, for calculating particular quantities, by the aid of the sliding rule.

METHOD OF PERFORMING CALCULATIONS BY THE SLIDING RULE.

THIS instrument is a mechanical application of logarithms; and to have a correct idea of its principle of action, we must consider the operation of logarithms, whereby they perform the multiplication and division of numbers.

LOGARITHMS are a series of artificial numbers, adapted in a particular manner to a series of real numbers, and arranged in a table, wherein every real number has its corresponding logarithm; so that by inspection of such a table, any number can be converted into its logarithmic representative; and conversely any logarithm can be converted into the real number which it represents (*a*).

To multiply any two numbers together, by the aid of a table of logarithms, we must substitute the logarithm of each number, for the number itself, and then add the two logarithms together; their sum will be another logarithm, which being reconverted by the table, into a real number, that number will be the product of the two original numbers, which have in effect been multiplied together, by this addition of their logarithmic representatives.

And conversely, the division of one number by another, can be effected by subtracting the logarithm of the divisor, from the logarithm of the dividend, and the remainder is the logarithm of the quotient.

Hence, logarithms tend to facilitate computations, by substituting the operations of addition and substraction, for those of multiplication and division, which are more tedious and difficult to be performed.

Logarithms were first invented by John Napier, Baron of Merchiston, in Scotland, who published an account of his discovery in Latin, intitled *Mirifici Logarithmorum Canonis descriptio*, 1614. The numbers given in the inventor's table were Hyperbolic Logarithms, see p. 345; but the tables of common loga-

(*a*) *The logarithm of a number is the index of that power of ten which will produce the number.* Hence the logarithm is one more, than the number of times that ten must be multiplied by itself, to produce the number which the logarithm represents.

Example. The logarithm of 100 is 2, because 100 is the 2nd power (or square) of 10; for, as 10 must be multiplied once by 10 to produce 100, the logarithm of 100 is $1 + 1 = 2$. And in like manner, 1000 being the third power (or cube) of 10, its logarithm is 3; for ten must be multiplied twice by itself to produce 1000, and $2 + 1 = 3$.

It follows from this construction of logarithms, that they will form a series which increases in arithmetical progression, whilst the numbers which they represent, form a series which increases in geometrical progression; and the two series are so adapted to each other, that 0 in the arithmetical series corresponds to 1 in the geometrical series.

Thus {	0	1	2	3	4	5	&c.	Logarithms forming an arithmetical series.
	1	10	100	1000	10000	100 000	&c.	Numbers forming a geometrical series.

This is the skeleton of a table of logarithms, for all the intermediate number between 1 and 10; 10 and 100, &c., in the geometrical series, may be filled up, and may have logarithms properly proportioned to them, to fill up the corresponding intervals between 0 and 1; 1 and 2, &c., in the arithmetical series.

It is a consequence of this construction, that the logarithms of all numbers which are less than 10 will be decimal fractions; for instance, the log. of 5 is 0·69897. And the logarithms of the numbers between 10 and 100, will be 1, with certain decimal fractions in addition; example, the log. of 50 is 1·69897. And the logarithms of numbers from 100 to 1000, will be 2, with the addition of suitable decimals; for instance, the log. of 500 is 2·69897.

rithms now generally used, were deduced from Napier's, by Mr. Henry Briggs in 1615, and for a long time they were called Briggs's logarithms.

The following table contains the logarithms of every number from 1 to 100, but the decimal fractions of the logarithmic numbers are only carried to three places of figures, because the divisions of sliding rules are not commonly made to represent more minute quantities than thousandth parts of the whole scale.

Num.	Logar.	Num.	Logar.	Num.	Logar.	Num.	Logar.	Num.	Logar.
1	0·000	21	1·322	41	1·613	61	1·785	81	1·908
2	0·301	22	1·342	42	1·623	62	1·792	82	1·914
3	0·477	23	1·362	43	1·633	63	1·799	83	1·919
4	0·602	24	1·380	44	1·643	64	1·806	84	1·924
5	0·699	25	1·398	45	1·653	65	1·813	85	1·929
6	0·778	26	1·415	46	1·663	66	1·820	86	1·934
7	0·845	27	1·431	47	1·672	67	1·826	87	1·940
8	0·903	28	1·447	48	1·681	68	1·833	88	1·944
9	0·954	29	1·462	49	1·690	69	1·839	89	1·949
10	1·000	30	1·477	50	1·699	70	1·845	90	1·954
11	1·041	31	1·491	51	1·708	71	1·851	91	1·959
12	1·079	32	1·505	52	1·716	72	1·857	92	1·964
13	1·114	33	1·519	53	1·724	73	1·863	93	1·968
14	1·146	34	1·531	54	1·732	74	1·869	94	1·973
15	1·176	35	1·544	55	1·740	75	1·875	95	1·978
16	1·204	36	1·556	56	1·748	76	1·881	96	1·982
17	1·230	37	1·568	57	1·756	77	1·886	97	1·987
18	1·255	38	1·580	58	1·763	78	1·892	98	1·991
19	1·279	39	1·591	59	1·771	79	1·898	99	1·996
20	1·301	40	1·602	60	1·778	80	1·903	100	2·000

Examples of the use of logarithms. To multiply 16 by 4. The log. of 16 is per table 1·204; add thereto the log. of 4, which is 0·602, and we have 1·806 for the sum of the two logarithms. If we seek this amongst the logarithms in the table, we find its corresponding number is 64; which is the product of the two numbers 16 × 4.

To divide 96 by 8. Take the log. of 96, which is 1·982; deduct from it the log. of 8, which is 0·903, and the logarithm remainder is 1·079; according to the table, this difference of the two logarithms is the logarithm of the number 12; which is the quotient of 96 ÷ 8.

The most important use of logarithms is to abridge those complicated multiplications and divisions, which are required to perform involution, or the raising of the powers of numbers; and evolution, or the extraction of their roots. For instance, to obtain the square of a number, it must be multiplied by itself; but if we multiply its logarithm by 2, the product will be the logarithm of its square. And conversely, if we divide the logarithm of any number by 2, the quotient will be the logarithm of its square root. In like manner, by multiplying or dividing the logarithm of a number by 3, we obtain the logarithm of its cube, or of its cube root. And any other power, or root, of a number may be raised or extracted by multiplying, or dividing, the logarithm of the number by the index of the power in question.

Examples. To extract the cube root of 64. Its log. is 1·806, which being divided by 3, gives 0·602, which is the log. of 4, the root required. Again, to extract the square root of 81. Divide its log. 1·908 by 2, and we have 0·954, which is the log. of 9, the root sought. Or to obtain the square of 5, multiply its log. 0·699 by 2, and we obtain 1·398, which is the log. of 25, the square demanded.

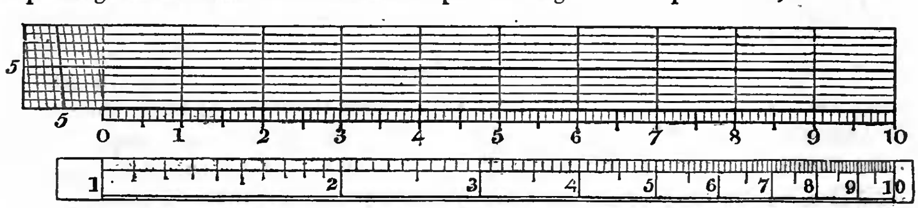
The above is a sufficient statement of the properties of logarithmic numbers, to explain the construction and operation of the sliding rule, which consists of a combination of straight lines, engraved on the edges of rulers, and graduated with unequal divisions, in such manner, that the magnitudes of the spaces or distances of each of the divisions, from the first division of the scale, will represent the series of logarithmic quantities, not by numbers, as in the table, but by spaces. And the divisions are figured, so as to denote the real numbers, corresponding to the logarithms which are represented by those spaces.

Hence in the ordinary use of logarithms, we substitute for the real numbers, certain artificial logarithmic numbers, as their representatives; and in the sliding rule we make a still farther substitution, viz. that of spaces, to represent these logarithmic numbers, which are themselves only artificial quantities.

At first sight this would appear to be a complication of the logarithmic method of computation; but in effect it will be found to be a great simplification, because it divests that system of all idea of number, in reference to the logarithms; for those numbers, which as logarithms are purely artificial quantities, become realities as spaces; also when the logarithmic quantities are represented by spaces, we can very conveniently perform the addition or subtraction of the logarithms, by applying those spaces in actual contact with each other, and joining one space to another, so as to obtain their sum, in order to perform multiplication; or cutting off from one space, a quantity equal to another space, so as to obtain their difference, in order to perform division.

To divide a line into logarithmic spaces, we must provide a plane scale of any suitable length, which is accurately divided into 10 equal parts, with decimal subdivisions at one end, and with diagonal lines, to obtain such minute subdivisions as will represent thousandths of the whole length. From this scale we must measure off with a pair of compasses, such a space from the zero of the scale as will represent any logarithm in the table; and the space so measured must be transferred to the line which we intend to divide logarithmetically, by placing one point of the compasses upon the first division or commencement of that line, and marking a division upon it, at the proper place, with the other point of the compasses.

Note. In thus forming a logarithmic scale, we must disregard the whole numbers which constitute a part of the logarithms in the table, and only consider the decimals; whereby we assume that all the logarithmic numbers are decimal fractions of unity; and therefore to represent them, we take corresponding decimal fractions of the whole space or length on the plain scale, from 0 to 10 (*a*).



Plane scale with equal divisions.

Logarithmic scale with unequal divisions.

(*a*) This method of expunging the whole numbers of the logarithms is not peculiar to the sliding rule, but it is done in all modern tables of logarithms; those whole numbers are called the indices of the logarithms to which they belong; and the indices are omitted in the tables, because they can be very readily supplied, being in all cases one less than the number of places of figures contained in the whole number which is to be represented by the logarithm, when that index is prefixed to it.

For instance, the logarithm of 4 is 0·60206; and that of 40 is 1·60206; or of 400, 2·60206; or of 4000, 3·60206. In all modern tables the logarithm ·60206 is given for 4 every time it recurs, as at 40, 400, &c. without regard to the number of 0's which follow it; and the index, or whole number, which is to be prefixed to the logarithm, is omitted in the table; but it must in all cases be supplied, to the logarithms of any numbers which are taken out from the table for use.

For example, to set out the primary or figured divisions of the logarithmic line, we must successively transfer from the plane scale spaces of 301, 477, 602, 699, &c. as per table; that is to say, the logarithms of 2, 3, 4, &c. must be represented by 301, 477, 602, &c. thousandths of the whole length of the plane scale from 0 to 10; and those spaces must be marked off respectively from the first division of the logarithmic line, which is numbered 1.

The primary divisions thus obtained, may be numbered with figures 1, 2, 3, 4, 5, &c. to 10, and these figures will denote real numbers (either whole numbers or decimal fractions) whose logarithms are expressed by the spaces or distances at which they are placed respectively from the first division marked 1; and all those logarithms are considered as decimal fractions of the whole space from 1 to 10, which is taken for the unity of the system, and is called its radius. By continuing the same operation of measuring off the intermediate logarithms from the plane scale, and transferring those measures to the logarithmic scale, all the intervals between the primary divisions may be filled up, and the scale completed, as in the figure. Or, by reference to a more extensive table of logarithms, the scale may be filled up with as many other subdivisions as it will admit of, without becoming too crowded with minute divisions, to leave them distinguishable by the eye.

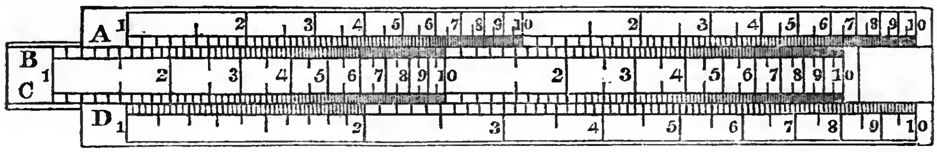
The simple scale, or line, with logarithmic divisions, is called Gunter's scale, from the name of the celebrated mathematician who invented it about 1623. The line contains two series of the logarithmic divisions from 1 to 10, placed one at the end of the other in continuation, to form one line, which is used with the compasses, to perform the multiplication and division of numbers, and the evolution or involution of powers, and roots of numbers. Thus the space from the first division 1 of the scale, to the division representing any number which is to be multiplied (16 for instance) being taken in the compasses; we can set off that space from the division representing the other number, or multiplier (4 for instance) measuring forwards in the direction from 1 towards 10; and thus prolonging one space by adding the other to it, we shall find the other point of the compasses will reach to some further division (64 in this case) which represents the product of the multiplication (of 16 by 4).

Or to divide any number (as 96) by another number (as 8), take the space in the compasses from 1 to the division representing the divisor (8); and set off that space from the number to be divided (96) measuring backwards from 10 towards 1; then the other point of the compasses will fall on a division (12) representing the quotient obtained by dividing the large number (96) by the other (8).

The simple Gunter's scale was improved by Mr. Wingate in 1627, who formed it on two separate rulers, which were applied one against the other, to avoid using compasses. This was modified into the present sliding rule by a Mr. Milburne in 1650, and by Seth Partridge in 1657, it was for a long time called the sliding Gunter. This instrument consists of a ruler, having a moveable slider fitted into a groove along the middle of the ruler; and the adjacent edges of both the groove and the slider being graduated with logarithmic divisions, the spaces representing the logarithms can be added to, or subtracted from each other, by comparison of contact, without using compasses; and when the slider is properly placed, the results of the calculation are obtained by mere inspection.

The lines upon sliding rules have been combined in various forms, to suit the purposes of particular calculators; but that which has been found most convenient for the use of engineers, was arranged by Mr. Southern, under the direction of Mr. Watt, expressly for the use of the engineers of Messrs. Boulton and Watt's manufactory at Soho, and is in consequence called the Soho rule. The original divisions for these instruments were made with the greatest accuracy, the radius of the scale being 10 inches long; but similar lines have since been divided, with equal accuracy, on a larger scale; and the divisions being less minute, they are more easily read off, and admit of more minute subdivisions.

DESCRIPTION OF THE SOHO SLIDING RULE. The rule itself is made of hard box-wood, $10\frac{1}{2}$ inches long, $\frac{8}{10}$ broad, and about $\frac{2}{10}$ thick. A groove is formed along the middle of one side of it; and a slider of the same wood, $\frac{3}{10}$ wide by $\frac{1}{10}$ thick, is fitted into the groove, so as to slide freely endways, backwards or forwards therein.



The face of the slider, and that of the ruler itself, are reduced to one flat surface, upon which the divisions and figures are engraved, as is represented in the sketch. There are four lines, divided logarithmically, viz. two on the fixed ruler, at the margins of the groove, and two others on the margins of the moveable slider; so that there are two pairs of lines, each pair consisting of one moveable, and one fixed line of divisions.

The several lines are designated by the four first letters of the alphabet; the two upper lines are marked A and B, and the two lower lines are marked C and D. The lines A, B and C are all alike, being fac similes of each other, but the lower line D has divisions of exactly twice the magnitude of the others; this lower line, which is called a line of single radius, is formed as before described, and the space from 1 to 10, which is called the radius, occupies 10 inches of the length of the rule. Each of the upper lines contains two repetitions of the series of smaller logarithmic divisions, each series being 5 inches radius; these are called lines of double radius, but it would be more correct to say lines of two radii, and to call line D a line of double radius.

NUMERATION, OR NOTATION, ON THE SLIDING RULE. The first step to learning the use of this valuable instrument, is to acquire a facility in reading its divisions, so as to find that division which represents any required number. It should be premised, that the value of the divisions on the logarithmic lines is not permanently fixed; but that an arbitrary value is given to them by the calculator in each particular operation, so that the ten primary figured divisions of the scale may equally represent 1, 2, 3, 4, &c.; or 10, 20, 30, 40, &c.; or 100, 200, 300, 400, &c.; or ·1, ·2, ·3, ·4, &c., according to the assumption made in each case.

Hence the sliding rule does not determine the numbers which result from the calculations which are made by it, but only the figures by which the numbers are to be denoted, and the order in which those figures are to stand; but their actual value must be determined by other means.

The reason of this deficiency will be evident, when we reflect that in forming the logarithmic scale, all the whole numbers appertaining to the logarithms were rejected, and only the decimal portion of each logarithm retained, to be represented by the spaces of the divisions. These whole numbers indicate the exact value of the numbers represented by the logarithms; and hence the whole numbers are called the indexes or characteristics of the logarithms to which they are prefixed.

By altering the index of the logarithm of any number, it will become the logarithm of any other number which can result from multiplying or dividing the original number, by 10, or 100, or 1000, &c. For instance, the log. of 2·5 is 0·39794; and of 25, 1·39794; and of 250, 2·39794; and of 2500, 3·39794; so that, by applying a suitable index, the same decimal portion of the logarithm will serve for all numbers which can be denoted by 2, followed by 5. The sliding rule, as commonly constructed, has nothing to represent the index or whole number of the logarithms, and, consequently, its divisions cannot have a determinate value.

In general, whatever value is assumed for the first division of a logarithmic line, whether ·1 or 1·, or 10 or 100, the other primary or figured divisions of the same line should preserve an uniform progressive value, according to their respective places in the scale; so that if the first division is accounted ·1, the next will be ·2, then ·3 and ·4, &c.; but the first division being called 1, then the others will be 2, 3, 4, &c. Or if the first is assumed to be 10, the others will be 20, 30, 40, &c.; or the first being 100, we shall have 200, 300, 400, &c. for the others.

All intermediate subdivisions must be reckoned according to the value that has been assumed for the primary divisions between which they are situated.

Note. At the same time that we ought thus to preserve an uniformity of value, in the progressive divisions of the same line, we may, at pleasure, assume different values for the divisions of the different lines on the same rule, even when they are used in concert for one operation.

We may now explain the notation by some examples, which the reader must perform with the rule itself, as the sketches being small and immovable, are not sufficiently explanatory to a learner (*a*). It is easy to find the division which will represent any number which can be expressed by one figure with any number of 0's (as ·6, or 6, or 60, or 600,) because the primary divisions of the scale are figured suitably for that purpose.

To find numbers of more than one place of figures on the sliding rule, we must keep the following facts in mind: The primary figured divisions, represent the first or left hand figures of the numbers. The subdivisions between the primary divisions, represent the figures in the second place from the left towards the right hand. The intermediate divisions between these subdivisions, represent the figures in the third place from the left. In some long rules the intermediate divisions are again partially subdivided, to represent some of the figures in the fourth place. Lastly, in all cases when the subdivisions and intermediate divisions required to express any number, are not engraved on the scale, they must be imagined, by estimating the intermediate spaces by the eye.

Example. To find any number of two figures (for instance 64, or the line A), we must begin with the first or left hand figure (6), and select that primary division which is marked with the same figure (6). Then we must take the second figure towards the left hand (4), and find its corresponding subdivision, between the primary division last found (6) and the next beyond it (7). There are always ten of these subdivisions engraved between every two primary divisions, in the shortest rules, and the middle subdivision is drawn longer than the others, in order to distinguish it for five (and in some long rules it is figured with a small 5); taking this long subdivision for a guide, it is easy to acquire the habit of counting all the ten subdivisions, as quickly as if each one were figured; thus 64 will be found at the short subdivision adjacent to 65, on the side towards 60. For numbers which are denoted by only two figures, we shall always find a subdivision engraved to represent the second or last figure, and we shall have no occasion to imagine intermediate divisions, or to estimate any spaces by the eye. The sketch of the rule shows its slider drawn out so far, at the left hand, that the middle division of the line B (figured 10) is opposite to 64 on A, and serves as a pointer thereto. Also 5, on line B, points out 32 on line A; again, 4 on line D points out 25 on line C.

Example. To find a number of three figures (for instance 1·56 on the line B). The first figure (1) is represented by the primary division figured 1, at the commencement of the line B; then for the second figure (5), we must look amongst the subdivisions between the primary divisions (1 and 2). The fifth of those subdivisions is marked by a long stroke, to represent 1·5, and the short subdivision which follows it is 1·6; therefore the number we seek must be between those two; and if there are no intermediate graduations between the subdivisions, we must estimate $\frac{6}{10}$, or a little more than half the space between the subdivisions 1·5 and 1·6 to fix the place of 1·56. In the sketch of the rule 1·56,

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(*a*) Mr. Bate, optician in the Poultry, London, has taken great pains in dividing correct logarithmic scales for sliding rules, and has brought the manufacture of those instruments to the highest perfection; they are made by him in ivory and in box-wood, of various dimensions, suitable for different purposes. The original Soho rule, of 10 inches radius, is a very convenient size for the pocket; but they are also made of 24 inches radius, which is preferable for an office.

The calculations in this work have been made with a sliding rule, of nearly 28 inches radius, which was made by Mr. Bate for the author, and is divided with extreme precision. In correcting the impression, the same calculations have been repeated with another of Mr. Bate's sliding rules, of double the length of the former, or 56 inches radius. All engineers ought to be provided with such instruments, and with a little previous study, to become familiar with the notation; they will be enabled to calculate the proportions of machines and engines with the utmost facility, by the aid of the formulæ laid down in this work.

upon the line B, is opposite to 1 on line A, whereby the latter division becomes a pointer to the former. Also 10, at the middle of the line C, will point out to the student 2·53, or 253 on line D.

Note. Learners are liable to mistakes in reading off numbers of three or more figures, when the second figure is an 0 (such as 105 or 401), and it requires some attention to avoid taking 150 or 410 instead. It may be kept in mind, as a guide in these cases, that the divisions representing numbers which have an 0 in the second place of figures, will in all cases be found close to, or very near to, a primary figured division.

In some parts of the scales of short lines, the subdivisions which represent the second place of figures, are too near together to admit of engraving ten intermediate divisions between each of them; in such places five minute divisions are sometimes inserted, to represent the third place of figures; and therefore in reading such intermediate divisions, each one must be counted as two units of the third place of figures; the five divisions will therefore represent the even units, viz. 2, 4, 6, and 8; and consequently when the odd units 1, 3, 5, 7, or 9, occur in the third place of figures, the spaces between the five minute divisions must be subdivided by estimation.

Again, in other parts of the scale, the spaces between the subdivisions, which repreesnt the second place of figures, will only admit one intermediate division to be inserted; it will therefore represent five units of the third place of figures, and care must be taken to count it as such.

Hence, on the same rule, we shall find, at different parts of its scale, three kinds of minute intermediate divisions engraved between the larger subdivisions, in order to fill up and represent the third place of figures. At the commencement of the scale the interval between the subdivisions is filled up complete, with ten intermediate divisions; each division will then represent an unit in the third place of figures. In the middle parts of the scale only five intermediate divisions are inserted; and then each one will represent two units in the third place of figures. And in the higher parts of the scale, only one intermediate division is engraved; and it will represent five units in the third place of figures.

It is needless to give any other examples of this kind, as nothing but practice with the rule itself, can give a facility in reading off its divisions, and in estimating the exact places for those small intermediate divisions, for the third place of figures which are not engraved; and a considerable practice is requisite to acquire the habit of reading quick and correct. It is best to begin with a small Soho rule of 10 inches radius, and first practise as above, with the aid of the sketch and directions, to find out numbers of only two figures; then proceed to seek for numbers of three figures, which will require some estimation for the third place of figures, except at the commencement of the lower line D, which being a 10 inch radius, the intermediate divisions for each number, in the third place of figures, are inserted.

Having acquired the habit of reading the small rule, the learner should then proceed to practise with a larger one, in which the intermediate divisions are more numerous, and the spaces larger, so as to admit of more accuracy in estimating the last place of figures by the eye. Two kinds of long sliding rules have been divided by Mr. Bate, one being 28 inches radius, and the other 56 inches radius. The latter has divisions at the commencement of its scale, which represent every other number in the fourth place of figures, without estimation; and the middle part contains every division for the third place of figures; and the upper part of the scale every other division for the third place of figures.

Multiplication by the sliding rule. This is performed by the two similar lines marked A and B. The division representing one of the factors, or numbers to be multiplied, being found on the line B, the slider must be drawn out until that division is brought opposite to 1 on the line A. Then the division representing the other factor, or multiplier, being found on the line A; the product of the multiplication, will be found opposite to it, upon the line B; see the sketch, and also the following precept.

Sliding Rule.				*Example.*			
	A	1	Multiplier.		A	1	45
	B	Multiplicand.	Product.		B	1·56	70·2

This mode of stating the precept and example, is an exact representation of the manner of placing the slider, and of seeking the coincidences of the divisions upon the adjacent lines. The two letters at the commencement, denote the lines which are to be used. Mr. Watt and Mr. Southern used this form of stating theorems, and it is followed by the engineers of their school; it has also been adopted by Mr. Bevan in his treatise on the sliding rule.

When the slider is thus set, the rule forms a complete table of products of the multiplicand, into any multiplier, we choose to select on the line B, thus

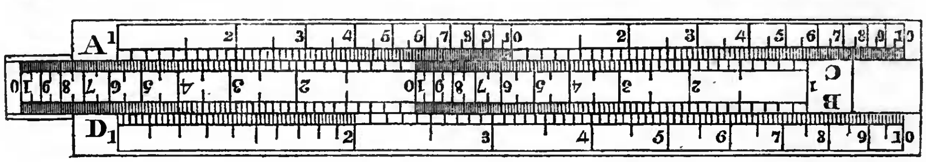
Exam.	A	1	2	3	4	5	6	7	8	9
	B	1·56	3·12	4·68	6·24	7·80	9·36	10·92	12·48	14·04

Note. As the lines A and B are divided exactly alike, it is immaterial upon which line we choose the multiplicand, and on which the multiplier, only observing that the two numbers to be multiplied, must in all cases be chosen on two different lines, and never both on the same line: also, that the product must be sought on the opposite line to that whereof the first division or 1 is used. A constant attention to this circumstance is indispensable, for if we depart from the precept, and read off from the wrong line, the operation of the rule will be to divide instead of to multiply, so as to give us a quotient where we required a product. The following precept gives the same results as the preceding:

Sliding Rule.	A	Multiplicand.	Product.	*Example.*	A	3·14	22	or	11
	B	1	Multiplier.		B	1	7		3·5

When we wish to use the sliding rule as a table, to obtain the various products of the multiplicand by different multipliers, we must place the multiplicand or factor, which is not to be altered, opposite to 1, and seek the different multipliers on the same line as the 1; then the products will be found opposite to those multipliers, on the same line as the multiplicand.

The sliding rule may have its slide inverted, and, in many cases, this is the most convenient mode of performing calculations by it. Thus draw the slider quite out of its groove, then turn it end for end, and slide it back into the groove, so that it will be inverted and reversed, as in this sketch, and the divisions on the slider must be counted backwards, in a reversed order to those of the direct line upon the ruler against which they apply.



This condition of the sliding rule is always marked *slide inverted* in the precepts.

Multiplication by the lines A *and* Ↄ *with the slide inverted.* Move the slider, so as to bring the two factors, or numbers which are to be multiplied together, opposite to each other, and then the product of their multiplication will be found opposite to 1 or 10; thus—

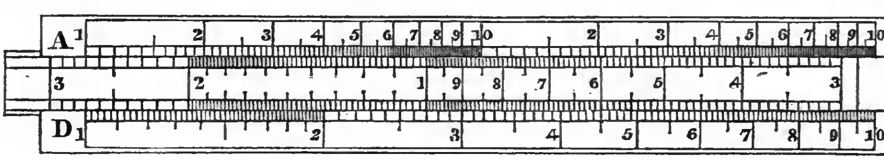
Sliding Rule, slide inverted.	A	Multiplier.	1	*Example.*	A	3·14	1	or	9	1
	Ↄ	Multiplicand.	Product.		Ↄ	7	22		3	27

It requires some practice to acquire the habit of reading off the divisions of the slide when inverted, but no new instructions can be wanted. On this plan, we must move the slider, so as to set the rule properly for each multiplication, and therefore it cannot form a table for the products of different multipliers, but the lines A and Ↄ form a table of all the different factors or pairs of numbers, which, being multiplied together, will give the same product.

In most cases of multiplication, the inverted slider will be preferable to the direct slider, because it will not admit of mistakes in reading off the results, and it is immaterial which of the two lines we use; for wherever we can find the two numbers which are to be multiplied together, we may place them opposite to each other, by moving the slider one way or the other; and the rule being so set, then wherever we can find 1 or 10, the product will be opposite to it. There will be more than one coincidence of the same numbers; but as they will all give the same result, we may examine them all, so as to obtain a correct result, independently of any small inaccuracy in the divisions of the rule.

Suppose, for instance, that 23 is to be multiplied by 25; the inverted slider must be set as in the sketch, with 23 on one line, opposite to 25 on the other line. On examination of the two lines, we shall find four different places where 23 on one line corresponds to 25 on the other line, and if the rule is accurately divided, those coincidences will be exact; or if there is any inaccuracy, it will be apparent, and the slider may be adjusted so as to obtain a mean of the errors, by making the coincidence at one place as exact as that at another. The slide being thus set, the product 575 will be found opposite 1, and opposite 10, so that there will be four places on the two lines A and C where this result may be found, and which may be compared together, to obtain a mean of the errors of the divisions.

THE INVERTED SLIDING RULE has two logarithmic lines, and is constructed expressly to be used in the manner before described. It may be formed by providing an extra slider to put into the groove of the Soho rule; such new slider being divided with a line of single radius, answering to that on the line D; but inverted or reversed in respect to it, and so arranged that the divisions begin at 3 instead of 1, and proceed to 10, which is situated near the middle of the length, and thence the divisions are continued onwards to 3, which forms the other end of the slider. This is called a broken inverted line of single radius. The figures to the divisions being engraved erect, it will be easier to read than the preceding.



In this way, two lines of single radius are brought into use instead of two of double radius, and, consequently, in a rule of given length, the divisions are twice as large. In this form of the inverted slider, we must use the two lower lines for simple multiplications and divisions. In the sketch, the slider is represented as set with 3 opposite 9, and we may trace two such coincidences; the product of this multiplication 27 is found opposite to 1, and there are two of those coincidences, both giving the same result.

The inverted rule is very convenient for performing all the ordinary processes of arithmetic, and is more accurate than the direct sliding rule, both by avoiding mistakes in reading, and by having divisions of double size, upon a rule of the same length. The inverted rule of 56 inches radius, which is made by Mr. Bate, gives very accurate results, and the sub-divisions being figured with small figures, in addition to the large figures which mark the primary divisions, it is extremely easy to find any number upon it.

In reading off the divisions of the inverted line on the slider, it must always be kept in mind, that the order of the numbers is reversed; so that they proceed from the right hand towards the left, contrary to the order of the direct line which counts from left to right. It would be more proper to call it the reversed slider, instead of the inverted slider; but the term has been adopted from the common sliding rule, when its slide is inverted as before described. The object of arranging the inverted line on the slider, with 1 near the middle of the slider, and 3 at each end, is to avoid the necessity of ever drawing the slider any farther out of its groove, than is absolutely necessary; and it will be found, on trial, that it is never required to draw out the slider much more than half its length at one end, or the other end, for then 1 at the middle of the slide comes opposite to 1 at either end of the rule; and this extent of motion brings the whole range of numbers into action (*a*).

Directions for moving the slider of the inverted sliding rule. In all cases of setting a slide rule, two numbers are given, one of which must be found upon the divisions of the rule, and the other upon those of the slider; and the slider must be placed so that those two numbers will correspond with each other. As there is only one series of numbers upon the slider of the inverted rule, and another series upon the rule, without any repetitions of the same numbers on each line, it is necessary in every case to draw out the slider, either at one end of the rule or at the other end, according to the following directions; for if the slider is drawn out at the wrong end, the required coincidences will not appear amongst the divisions of the rule and those of the slider.

In sliding rules which have the broken inverted line of divisions engraved upon the rule (*b*): Multiply the first figure of each of the two numbers together, and if the first figure of their product is more than 3, then draw out the slider at the left hand end of the rule; this case will occur most frequently. Or if the first figure of the product is less than 3, then draw out the slider at the right hand end of the rule. For instance: to set 7 to 9, the first figure of their product (63) is 6, which being more than 3, the slider must be drawn out at the left hand end, until 7 on one line of divisions is observed to correspond with 9 on the other. Again: to set 10 to 12, the first figure of their product (120) is 1, which is less than 3, therefore the slider must be drawn out at the right hand end, until 10 on one line of divisions is brought to correspond with 12 on the other line. These directions should be preserved in the memory, until it becomes a habit to move the slide the right way without any thought.

In order to set the slider with expedition, when it is decided which way to move it, both the required numbers (or their nearest whole numbers) should be glanced at with the eye, one number on the rule and the other on the slide, so as to contemplate the distance between them, and then the slider may be moved one way or other over that distance with rapidity, in order to bring the two numbers near enough together, that they may both come within the field of vision, and be contemplated at once. The attention should then be steadily fixed upon the division or space between two divisions upon the rule, which represents one of the given numbers, and at the same time observing the divisions of the slider as they pass by it in succession, when the slider is moved slowly along in its groove. The motion given to the slider should not be more rapid than will allow the eye to recognize the different divisions as they pass by; and when the required di-

(*a*) The use of the sliding rule, with its slide inverted, was first proposed by the Rev. Wm. Pearson, in 1797, in Nicholson's Journal, quarto, vol. i., p. 450; and the author adopted the method from reading that paper. The complete inverted, or more properly, the reversed slide rule, was first arranged by Dr. Wollaston, for the purpose of calculating chemical equivalents, and other similar calculations. An inverted sliding rule, of a short length for the pocket, was afterwards arranged by Mr. John Taylor, with a small printed table of directions for the use of it; but it is not yet brought into extensive use, nor is it explained in any book on the sliding rule.

(*b*) In sliding rules which have the broken inverted line of divisions engraved upon the slider, as is shown in p. 541, then the directions must be reverse to the above; for right hand read left hand, and vice versa.

vision, or space between two divisions, on the slider comes in sight, the motion should be so much retarded, as to enable the eye to count the several subdivisions which arrive in succession opposite to the number which has been chosen on the rule; and when the required coincidence is obtained, the slider is properly set.

Note. To attain this final adjustment of the subdivisions, the slider should always be moved in that direction which will cause the numbers to count upwards, as they pass in succession, and not downwards: if the inverted line of divisions is upon the rule, then the slider must be moved from right to left for that purpose; and therefore in cases when it is necessary to draw out the slider at the right hand end, it should be moved a little beyond the required coincidence in the first instance, and should then be returned slowly from left to right, to establish the exact coincidence.

Division by the sliding rule. This may be performed either with the direct slider, or with the inverted slider; and in either case it is the reverse of multiplication.

If the slider is direct, set it so that the divisor is opposite to 1; then seek the dividend on the same line as the divisor is upon, and the quotient will be found opposite to the dividend, upon the same line as the 1; thus,

Sliding Rule.				*Example.*			
	A	1	Quotient.		A	1	50
	B	Divisor.	Dividend.		B	1·56	78

Note. The rule when thus set forms a table of the several quotients which may be obtained by dividing different numbers by the same divisor.

With the slide inverted, the operation of division is very simple, thus; set the slider so that the number to be divided is opposite to 1 or 10, and then the quotient will be found opposite to the divisor. There are two coincidences for each reading, and they cannot be mistaken, for we may take either of the numbers on either of the lines, wherever they can be found.

Sliding Rule, slide inverted.				*Example.*				or		
	A	1	Quotient.		A	1	50		1	4·5
	Ɔ	Dividend.	Divisor.		Ɔ	780	156		27	6

Note. The rule when thus set forms a table of the several quotients which may be obtained by dividing the same number by different divisors.

Multiplication and division may be performed at one operation by the sliding rule. For the product which would result from the multiplying one number by another, may be divided by a third number, and the resulting quotient may be obtained, by inspection, with very great facility, because it is not necessary to observe the intermediate product. It is one of the great advantages of calculating by the sliding rule, that it is capable of abridging the arithmetical operations which it performs, by concentrating two or three operations into one, and exhibiting the final result at once, whereby the risk of errors is avoided, as well as the trouble of forming and recording intermediate products or quantities, which are commonly of no use in themselves, but only as stages of the process which must be gone through, in the usual mode of numerical computation.

When the slider is direct, one of the factors must be found upon one line A or B, and the divisor being found upon the other line, the slider must be set so as to bring them to correspond with each other; then the other factor being sought upon the same line as the divisor, the result will be found opposite to it, upon the same line as the factor first found.

Sliding Rule.					*Examp.*			
	A	Factor.	Result.	Quotient.		A	16 mltplier.	28·8 result.
	B	Divisor.	Factor.	1		B	25 divisor.	45 mltplcand.

The product of the multiplication cannot be found upon the rule, when the slide is direct, because it operates first to divide one of the factors by the divisor, and then to multiply the other factors by the quotient obtained by that division; thus 16 ÷ 25 = ·64 × 45 = 28·8, which is the result, the same as above. The intermediate quotient may, if required, be found upon the same line as the result, opposite to 1 upon the same line as the divisor.

When the slider is inverted it must be set so as to bring the two factors, or numbers, which are to be multiplied together, to correspond with each other; and then the divisor being sought upon one line, the result will be found opposite thereto, upon the other line. *Example.* To multiply 16 by 45, and then to divide their product by 25. Thus 45 × 16 = 720 ÷ 25 = 28·8 result.

Sliding Rule, slide inverted.	A	Factor.	Divisor.	*Example.*	A	45 factor.	25 divisor.
	Ɔ	Factor.	Result.		Ɔ	16 factor.	28·8 result.

The intermediate product of the multiplication need not be observed or attended to; but, if it is wanted, it may be found, by inspection, upon either of the lines opposite to 1 upon the other line.

The above operation is the same as that by which the rule of three or proportion is performed. For an example of the use of this property of the sliding rule, suppose that the dimensions of the two sides of a rectangular parallelogram are given in inches, and that it is required to find its area in square feet; the product obtained by multiplying the two sides of the rectangle together, will represent its area in square inches, which being divided by 144, the quotient will give the area in square feet. The rule must be set thus:

Sliding Rule, slide inverted.	A	Side of rectangle inc.	Divisor 144	*Examp.*	A	8 inc.	div. 144
	Ɔ	Side of rectangle inc.	Area sq. feet.		Ɔ	9 inc.	·5 sq. ft.

In like manner to obtain the area of a circle or an ellipsis in square feet, having the diameter, or the two diameters given in inches. The divisor for such cases will be 183·34, for that number of circular inches are equal to a square foot.

Sliding Rule, slide inverted.	A	Diameter inches.	Divisor 183·3	*Exam.*	A	43 inc. diam.	div. 183·3
	Ɔ	Diameter inches.	Area sq. feet.		Ɔ	64 inc. diam.	15 sq. ft.

Proportion, or the rule of three by the sliding rule. It is necessary to have a very precise idea of the method on which proportionate numbers are to be found by the rule of three, which is so called because three numbers are given, and the object of the calculation is to find such a fourth unknown number, as shall bear the same ratio to one of the three given numbers, as that which already exists between the other two given numbers. The two corresponding numbers between which the ratio is established, may be called *the terms of the ratio*, and, in all cases, they must both express different quantities of the same kind of thing. The other or third number, to which the unknown number is to be adapted, may be called the *known number*, and it will express a certain quantity, the same kind of thing, as that which the unknown number is required to express.

The problem is to find such a number for the fourth term, as will bear a certain ratio to the known number; that ratio is expressed by the two terms of the ratio, considered merely as relative or comparative numbers, without regard to their individual values as quantities. If one of the terms of the ratio is divided by the other, the quotient will show how many times one term is contained in the other, so as to represent the ratio which exists between them in the simplest manner; and then if the known number is multiplied by that quotient, the product will be the fourth number required; and which, from the manner by which it is obtained, will be so adapted to the known number, as to bear the same ratio to it, as that term of the ratio which was divided, bears to the other term of the ratio which was used for the divisor.

Hence, when the fourth number is obtained, there will be two pairs of numbers; the two numbers of each pair expressing the same kind of thing, and the ratio between them being the same in both pairs; for if the largest number of each pair is divided by the smallest, the quotient so obtained from each pair, will be the same number.

Note. It is immaterial whether the division or the multiplication is made first, for the known number may be multiplied by one of the terms of the ratio, and then the product being divided by the other term, the quotient will be the same number as would be obtained, by first dividing one term of the ratio by the other, and then multiplying the quotient by the known number, in the manner above stated.

The circumstances on which the proportion depends must be considered in each case, and it will be obvious, from the nature of the question, whether the unknown number ought to be greater, or less, than the known number to which it is to be adapted; that circumstance will determine whether it is a case of increasing or of diminishing proportion; and, accordingly, the smallest or the largest of the terms of the ratio must be taken for the divisor.

Rule of three questions may be considered as if the two terms of the ratio constituted the numerator and denominator of a vulgar fraction, by which the known number is to be multiplied, and the result will be the fourth or unknown number. Thus, instead of the usual expression, as 16 is to 12, so is 8 to 6; we may say ($\frac{12}{16}$ths of 8 is 6) twelve sixteenths of eight is six. Or instead of saying as 12 is to 16, so is 8 to $10\frac{2}{3}$; we may say ($\frac{16}{12}$ths of 8 is $10\frac{2}{3}$) sixteen twelfths of eight, is ten and two thirds.

Increasing proportion is when the unknown number ought to be larger than the known number to which it is to be adapted; for instance, as 12 is to 16, so is 8 to $10\frac{2}{3}$; or $\frac{16}{12}$ of 8 is $10\frac{2}{3}$. In such cases the smallest of the two terms of the ratio must be taken for the divisor, and the largest for the multiplier.

Diminishing proportion is when the unknown number ought to be smaller than the known number to which it is to be adapted; for instance, as 16 is to 12, so is 8 to 6; or $\frac{12}{16}$ths of 8 is 6. In such cases the largest of the two terms of the ratio must be taken for the divisor, and the smallest for the multiplier.

Example of increasing proportion. Suppose that the multiplying-wheel of a steam-engine has 72 teeth, and is turned round 19 times per minute, and that it actuates a pinion of 38 teeth upon the axes of the fly-wheel; query how many revolutions will that pinion and fly-wheel make per minute?

The terms of the ratio are 38 and 72, for those two numbers form one pair, and both represent the same kind of thing, viz. teeth; and the given ratio is that which exists between those two numbers. The known number is 19; it shows itself, because it has no correspondent, for want of the unknown number which is to be adapted to it, to complete the second pair of proportionate numbers: that unknown number will be the same kind of thing as the known number, viz. revolutions per minute. It is obvious that as the wheel is larger than the pinion, the pinion must turn round quicker than the wheel, therefore the unknown number will be greater than the known number 19, and this is a case of increasing proportion; consequently the smallest number of the two terms of the ratio (viz. 38) must be taken for the divisor.

The calculation may be as follows. 72 teeth $\div$ 38 teeth $=$ 1·895 times as many teeth are contained in the wheel as in the pinion; and therefore whilst the wheel makes 19 turns, the pinion must make 19 $\times$ 1·895 $=$ 36 turns per minute, which is the fourth number required. Or else the calculation may be, 72 teeth $\times$ 19 turns per minute $=$ 1368 teeth of the multiplying-wheel will act on those of the pinion in a minute; and as every 38 teeth which act, will cause one turn of the pinion, the latter must make (1368 $\div$ 38 $=$) 36 turns per minute, as before.

If we consider the two terms of the ratio as constituting the numerator and denominator of a vulgar fraction, by which the known number is to be multiplied, the unknown number which is sought, would be seventy-two thirty-eighths of 19, which is 36; for as $\frac{72}{38}$ths is equal to 1·895, it will be 1·895 times 19, which is $=$ 36 turns per minute.

Proportion may be performed by the sliding rule, either with the slider direct or inverted.

When the slider is direct, the largest term of the ratio must be found on one line, and the slider must be set with the smallest term upon the other line, in correspondence with the largest; then in cases of increasing proportion, the known number must be found upon the same line with the smallest term of the ratio, and the unknown number will be found opposite to it, on the same line with the largest term of the ratio. The known number must be sought upon the same line with that term of the ratio which is to be used as the divisor.

Sliding Rule.	A	Largest term of ratio.	Unknown number.	*Example.*	A	72 teeth.	36 turns.
	B	Smallest term of ratio.	Known number.		B	38 teeth.	19 turns.

The rule when thus set forms a complete table of all the possible pairs of numbers which have a common relation or analogy to each other; in the example, it is the ratio that 38 bears to 72; hence any two numbers which correspond on the lines A and B, would be proper for the number of teeth in the wheel, and in the pinion respectively; for instance, 108 teeth for the wheel, to 57 teeth in the pinion. Also any two corresponding numbers on the two lines, will represent the number of turns which the multiplying-wheel and the pinion must make in the same time; for instance, if the wheel made 24 turns per minute, the fly-wheel would make $45\frac{1}{2}$ turns per minute.

When the slider is inverted, the known number must be found upon one of the lines; and in cases of increasing proportion, the slider must be set with the largest of the two terms of the ratio in correspondence with the known number; then the smallest term of the ratio being found upon either of the lines, the unknown number will be opposite to it. The result or unknown number will be found opposite to that term of the ratio which is to be used for a divisor.

Sliding Rule, slide inverted.				*Exam.*			
	A	Known number.	Smallest term of ratio.		A	19 turns.	38 teeth.
	ɔ	Largest term of ratio.	Unknown number.		ɔ	72 teeth.	36 turns.

Example of diminishing proportion. Suppose that the multiplying-wheel has 72 teeth, and makes 19 turns per minute, and that the fly-wheel and pinion make 36 revolutions per minute; query how many teeth must there be in the pinion?

The terms of the ratio are 19 and 36; for those two numbers form one pair, and both represent revolutions per minute. The known number is 72 teeth, and requires the unknown number, which must represent the same kind of thing, viz. teeth. It is obvious that as the pinion revolves quicker than the wheel, the pinion must have fewer teeth than the wheel; so that the unknown number will be less than the known number; hence this is a case of diminishing proportion, and the largest term of the ratio (36) must be taken for the divisor.

Thus, 19 revolutions $\div$ 36 revolutions $=$ ·5278 of a revolution of the wheel is made, whilst the pinion makes one turn, and as the wheel has 72 teeth, the pinion must have 72 teeth $\times$ ·5278 $=$ 38 teeth.

Or 72 teeth $\times$ 19 revolutions $=$ 1368 teeth of the wheel act in a minute, and as they cause the pinion to make 36 turns per minute, it must have 1368 $\div$ 36 $=$ 38 teeth.

Or we may say that the number of teeth in the pinion must be ($\frac{19}{36}$ of 72 $=$ 38) nineteen thirty-sixths of seventy-two, which is thirty-eight.

Sliding Rule.				*Exam.*			
	A	Smallest term of ratio.	Unknown number.		A	19 turns.	38 teeth.
	B	Largest term of ratio.	Known number.		B	36 turns.	72 teeth.

The known number must be sought upon the same line with that term of the ratio which is to be used as the divisor. The rule when thus set forms a complete table of all the pairs of numbers which have the same ratio, as before explained.

Sliding Rule, slide inverted.				*Ex.*			
	A	Known number.	Unknown number.		A	72 teeth.	38 teeth.
	ɔ	Smallest term of ratio.	Largest term of ratio.		ɔ	19 turns.	36 turns.

When the slider is inverted, the result or unknown number, will be found opposite to that term of the ratio which is to be used for a divisor.

To convert vulgar fractions into decimal fractions for the sliding rule. All numbers which are to be used on the sliding rule must be according to the decimal notation, because the divisions and subdivisions of the logarithmic lines proceed by tens (*a*). According to the established customs of practical artists, quantities are subdivided into halves, thirds, quarters, eighths, twelfths, and sixteenths, and but few instances occur of fifths or tenths. For instance, a fathom is divided into half, to make a yard; the yard is divided into three, for feet; the foot into twelve, for inches; and each inch into halves, quarters, eighths, and sixteenths. The pound avoirdupois is divided into halves, quarters, and sixteenths.

This want of uniformity in the subdivision of weights and measures, and the want of correspondence in any of those subdivisions, with the established system

(*a*) Sliding rules have been made with the intermediate spaces between the primary divisions divided into twelfths, and those subdivided into quarters and eighths, conformably to the division of linear measures into feet and inches, and eighths of inches; others have been divided into eighths and sixteenths. Such rules are convenient for particular purposes, but are necessarily limited to those purposes; and none but decimal divisions can be recommended to engineers.

A series of tables of logarithms, in which the numbers proceed by fractions of twelfths, and by sixteenths, was published in 1817 by Mr. Thomas Preston, in a small octavo volume, entitled; a New System of Commercial Arithmetic, of such construction as to obviate all the inconveniences arising from the irregularity in the division of our moneys, weights, and measures. This is a useful book, and contains very explicit directions for the application of logarithms to common business.

of arithmetical notation, is a source of continual trouble in calculation; and to acquire a facility in the use of the sliding rule, it becomes necessary to retain in the memory, the decimal value of all those vulgar fractions which are in most common use, in the same manner as the multiplication table is learned.

The following table exhibits all these fractions. The twelfths in the table serve for converting inches into decimals of a foot; and the sixteenths for converting sixteenths of an inch into decimals of an inch; or ounces avoirdupois into decimals of a pound.

					$\frac{9}{12}$ ·75	$\frac{10}{12}$ ·8333	$\frac{11}{12}$ ·9166
$\frac{1}{2}$ = ·5							
$\frac{1}{3}$ = ·3333	$\frac{2}{3}$ ·6666						
$\frac{1}{4}$ = ·25	$\frac{2}{4}$ ·5	$\frac{3}{4}$ ·75					
$\frac{1}{5}$ = ·2	$\frac{2}{5}$ ·4	$\frac{3}{5}$ ·6	$\frac{4}{5}$ ·8				
$\frac{1}{6}$ = ·1666	$\frac{2}{6}$ ·3333	$\frac{3}{6}$ ·5	$\frac{4}{6}$ ·6666	$\frac{5}{6}$ ·8333			
$\frac{1}{8}$ = ·125	$\frac{2}{8}$ 25	$\frac{3}{8}$ ·375	$\frac{4}{8}$ ·5	$\frac{5}{8}$ ·625	$\frac{6}{8}$ ·75	$\frac{7}{8}$ ·875	
$\frac{1}{12}$ = ·0833	$\frac{2}{12}$ ·1666	$\frac{3}{12}$ ·25	$\frac{4}{12}$ ·3333	$\frac{5}{12}$ ·4166	$\frac{6}{12}$ ·5	$\frac{7}{12}$ ·5833	$\frac{8}{12}$ ·6666
$\frac{1}{16}$ = ·0625	$\frac{2}{16}$ ·125	$\frac{3}{16}$ ·1875	$\frac{4}{16}$ ·25	$\frac{5}{16}$ ·3125	$\frac{6}{16}$ ·375	$\frac{7}{16}$ ·4375	$\frac{8}{16}$ 5
$\frac{9}{16}$ = ·5625	$\frac{10}{16}$ ·625	$\frac{11}{16}$ ·6875	$\frac{12}{16}$ ·75	$\frac{13}{15}$ ·8125	$\frac{14}{16}$ ·875	$\frac{15}{16}$ ·9375	

When the decimal value of such vulgar fractions as occur, is not retained in the memory, they may be converted into decimals by the lines A and B of the sliding rule, either with the slider inverted or direct.

When the slider is direct, the numerator must be found on the line A, and the denominator on the line B, and the slider must be set so that they will correspond, and stand one over the other, in the same manner as fractions are usually written; the equivalent decimal may then be found on the line A, opposite to 1 on the line B; viz. on the same line with the denominator.

Sliding Rule. $\left\{\begin{array}{lll} \text{A} & \text{Numerator.} & \text{Decimal.} \\ \hline \text{B} & \text{Denominator.} & 1 \end{array}\right.$ *Examples.* $\begin{array}{lll} \text{A} & 3 & ·375 \\ \hline \text{B} & 8 & 1 \end{array}$ or $\begin{array}{ll} 13 & ·812 \\ \hline 16 & 1 \end{array}$

The rule when thus set forms a complete table, of all the possible fractions that are equivalent to the decimal number, which is opposite to 1 upon the same line with the denominators of the fractions.

When the slider is inverted, it must be set so that the numerator will correspond with 1, thus;

Sliding Rule, slide inverted. $\left\{\begin{array}{lll} \text{A} & \text{Numerator.} & \text{Decimal.} \\ \hline \text{Ɔ} & 1 & \text{Denominator.} \end{array}\right.$ *Examples.* $\begin{array}{lll} \text{A} & 3 & ·375 \\ \hline \text{Ɔ} & 1 & 8 \end{array}$ or $\begin{array}{ll} 13 & ·812 \\ \hline 1 & 16 \end{array}$

It is so frequently required to express inches, and parts of inches, in decimals of a foot, that the following tables will be found to save much time.

Decimal Value of Parts of an Inch.

Parts of an Inch.	Decimals of an Inch.	Decimals of a Foot.
$\frac{1}{32}$	·0312	·0026
$\frac{1}{16}$	·0625	·0052
$\frac{1}{8}$	·125	·0104
$\frac{1}{8}$ & $\frac{1}{16}$	·1875	·0156
$\frac{1}{4}$	·25	·0208
$\frac{1}{4}$ & $\frac{1}{16}$	·3125	·0260
$\frac{3}{8}$	·375	·0312
$\frac{3}{8}$ & $\frac{1}{16}$	·4375	·0365
$\frac{1}{2}$	·5	·0417
$\frac{1}{2}$ & $\frac{1}{16}$	·5625	·0469
$\frac{5}{8}$	·625	·0521
$\frac{5}{8}$ & $\frac{1}{16}$	·6875	·0573
$\frac{3}{4}$	·75	·0625
$\frac{3}{4}$ & $\frac{1}{16}$	·8125	·0677
$\frac{7}{8}$	·875	·0729
$\frac{7}{8}$ & $\frac{1}{16}$	·9375	·0781

Table of Inches and fractional Parts, expressed in Decimals of a Foot.

Inches and parts.	Decimals of a Foot.	Inches and parts.	Decimals of a Foot.	Inches and parts.	Decimals of a Foot.
$\frac{1}{8}$	·0104	4	·3333	8	·6667
$\frac{1}{4}$	·0208	$4\frac{1}{4}$	·3542	$8\frac{1}{4}$	·6875
$\frac{1}{2}$	·0417	$4\frac{1}{2}$	·375	$8\frac{1}{2}$	·7083
$\frac{3}{4}$	·0625	$4\frac{3}{4}$	·3958	$8\frac{3}{4}$	·7292
1	·0833	5	·4167	9	·75
$1\frac{1}{4}$	·1042	$5\frac{1}{4}$	·4375	$9\frac{1}{4}$	·7708
$1\frac{1}{2}$	·125	$5\frac{1}{2}$	·4583	$9\frac{1}{2}$	·7917
$1\frac{3}{4}$	·1458	$5\frac{3}{4}$	·4792	$9\frac{3}{4}$	·8125
2	·1667	6	·5	10	·8333
$2\frac{1}{4}$	·1875	$6\frac{1}{4}$	·5208	$10\frac{1}{4}$	·8542
$2\frac{1}{2}$	·2083	$6\frac{1}{2}$	·5417	$10\frac{1}{2}$	·875
$2\frac{3}{4}$	·2292	$6\frac{3}{4}$	·5625	$10\frac{3}{4}$	·8959
3	·25	7	·5833	11	·9167
$3\frac{1}{4}$	·2708	$7\frac{1}{4}$	·6042	$11\frac{1}{4}$	·9375
$3\frac{1}{2}$	·2917	$7\frac{1}{2}$	·625	$11\frac{1}{2}$	·9583
$3\frac{3}{4}$	·3125	$7\frac{3}{4}$	·6458	$11\frac{3}{4}$	9792

To find the square of a number by the sliding rule. For this purpose the number must be multiplied by itself, so that it is only a case of multiplication, which may be performed either with the slide direct or inverted; thus,

Sliding Rule.	A	1	Number.	*Example.*	A	1	8
	B	Number.	Square.		B	8	64
Sliding Rule, slide inverted.	A	Number.	1	*Example.*	A	8	1
	Ɔ	Number.	Square.		Ɔ	8	64

It is most convenient to use the slide inverted to obtain the squares of numbers, and particularly in cases where the square of a number is required to be divided by another number.

For instance, the rule given at the bottom of p. 31, to find the height that a body must fall to acquire a given velocity; Divide the square of the velocity in feet per second by $64\frac{1}{3}$; the quotient is the height fallen in feet. This and other similar cases may be performed by the sliding rule.

Sliding Rule, slide inverted.	A	Veloc. ft. per second.	64·33 feet.	*Ex.*	A	30 ft. per sec.	64·33 feet.
	Ɔ	Veloc. ft. per second.	Height fallen ft.		Ɔ	30 ft. per sec.	14 ft. faln.
Sliding Rule, slide inverted.	A	Veloc. ft. per minute.	231600	*Ex.*	A	1800 ft. per min.	·2316
	Ɔ	Veloc. ft. per minute.	Height fallen ft.		Ɔ	1800 ft. per min.	14 ft. faln.
Sliding Rule, slide inverted.	A	Veloc. miles per hour.	29·9	*Ex.*	A	20·45 mil. per hr.	29·9
	Ɔ	Veloc. miles per hour.	Height fallen ft.		Ɔ	20·45 mil. per hr.	14 ft. faln.

To find the square root of any number by the sliding rule. This is a case of division, and the rule may be set with the slider inverted, so as to form a complete table of all the quotients which can be obtained by dividing the required number by different divisors; hence we can select, by inspection, such a divisor as will produce a quotient equal to itself; and that divisor and quotient will represent the square root of the number.

The slide being inverted, set it so that 1 upon one line will point to the required number upon the other line; the divisions of the two lines are then to be examined, to find two coincident numbers, which are both of the same value, and those numbers will be the square root required. *Note.* As the numbers on one line proceed in a contrary direction to those upon the other line, it is easy, by counting along them, to find when the same number on both lines meet; or if it happens that two divisions representing the same number do not meet, the coincident point must be within the space between those two divisions which are nearest to a coincidence; and as there is only one such point, there can be no danger of mistake in finding it.

Sliding Rule, slide inverted.	A	Number.	Root.	This is where the same num-	*Ex.*	A	64 numb.	8 root.
	Ɔ	1	Root.	bers on both lines meet.		Ɔ	1	8 root.

The rule thus set, forms a table of all the quotients which can be obtained by different divisors.

The above cases of squares and square roots, may be more conveniently performed by means of the lines marked C and D, which are laid down on the sliding rule, expressly for such purposes as will be explained; but the inverted rule deserves the preference in all cases to which it is applicable, because its divisions are double the size of those on the line C, upon a rule of the same length; and that is a great advantage, both for accuracy, and for the facility of reading off the quantities.

To find the reciprocals of numbers by the sliding rule. When 1 is divided by any number, the quotient will be a decimal fraction, which is termed the reciprocal of the number. The chief use of reciprocals is to enable us to substitute the operation of multiplication for that of division, or vice versa; because the same results may be obtained by multiplying a number by a decimal fraction, as by dividing it by the whole number to which that decimal fraction is reciprocal.

For instance, a cubic foot of water weighs 62·5 pounds; and to find the weight of any number of cubic feet of water in pounds, that number must be multiplied by 62·5. The reciprocal of 62·5 is (1 ÷ 62·5 = ·016; and if the number in question is divided by ·016, the quotient will be the same, as the product which would be obtained by multiplying it by 62·5.

Example. 14 cubic feet × 62·5 lbs. = 8750 lbs. Or 14 cubic feet ÷ ·016 = 8750.

In calculating by the sliding rule, it is very often advantageous to employ the reciprocals of numbers for divisors, instead of the numbers themselves for multipliers; or the reciprocals for multipliers, instead of the numbers for divisors. Suppose for instance, that two multiplications or divisions are required to be made by the sliding rule at one operation; it is capable of performing multiplication and division at one operation, as before explained; and therefore, by substituting the reciprocal of one of the multiplying numbers, and using that reciprocal as a divisor, or vice versa, the same result may be obtained by one operation of the rule, as by two multiplications or divisions.

Reciprocals are most conveniently found with the slide inverted; for if it is set so that 1 on one line, corresponds with one on the other line, then the two lines will form a complete table of numbers and their reciprocals opposite to them.

Sliding Rule, slide inverted.	A	1	Numbers.	*Examples.*	A	62·5 numb.	or 16	or 3
	Ɔ	1	Reciprocal.		Ɔ	·016 recipro.	·0625	·333.

To find the reciprocals of numbers by the sliding rule with the slide direct, it must be set as follows:

Sliding Rule.	A	1	Reciprocal.	*Examples.*	A	1	·016	or	1	·333
	B	Number.	1		B	62·5	1		3·	1.

USE OF THE LINE OF SINGLE RADIUS MARKED D ON THE SLIDING RULE. This line is used in concert with the line of double radius marked C, for performing such calculations as involve the squares of numbers, or their square roots; for by means of these two lines, a number may be either multiplied or divided by the square of another number, or by its square root.

It has been already stated, p. 534, that when the logarithm of any number is multiplied by 2, the product will be the logarithm of the square of that number; and conversely, if the logarithm of any number is divided by 2, the quotient will be the logarithm of the square root of that number.

The divisions upon the logarithmic line marked C, upon the slide rule, called the line of double radius, or the line of squares, are exactly half the size of the divisions upon the line D, which is called the line of single radius, or the line of roots. If the first divisions of each of those two lines are placed in correspondence, then every number upon the line C will have its square root opposite to it upon the line D; and conversely, every number upon the line D, will have its square opposite to it upon the line C; thus,

Sliding Rule.	C	1	Square	or	Number.	*Examples.*	C	1	2 numb.	or	16 squ.
	D	1	Number		Root.		D	1	1·414 rt.		4 numb.

Note. The rule being thus set forms a complete table of the squares and square roots of numbers.

To multiply the square of a number by another number. The number which is to be used for the multiplier being found on the line C, and placed opposite to 1 on the line D, the product of the multiplication will be found on C, opposite to the number which is to be squared for the multiplicand on D, thus,

Sliding Rule.	C	Multiplier.	Product of multiplication.	*Example.*	C	8 mult.	72 prod.
	D	1	Factor to be squared.		D	1	3 squ.

Or the same result may be obtained with the slide inverted, when it is set as follows:

Sliding Rule, slide inverted.	B	Multiplier.	Product.	*Example.*	ꓭ	8 mult.	72 pro.
	D	Factor to be squared.	1		D	3 squ.	1

This property of the lines C and D renders the sliding rule extremely convenient for computing the solidities of all prismatic, cylindrical, or spherical bodies. For the side of a square prism, or the diameter of a cylinder, being taken on the line D, as that factor which is to be squared, and the length of the prism or cylinder being taken for the other factor on the line C or 𝔤, then the solidity may be found on the line C or 𝔤, as follows, according as the slider is direct or inverted.

Sliding Rule.	C	Length of cylinder.	Solidity of cylinder.	*Exam.* C	4 ft. long.	16 cyl. ft.
	D	1	Diameter of cylinder.	D	1	2 ft. dia.

Sliding Rule, slide inverted.	𝔤	Length of cylinder.	Solidity of cylinder.	*Exam.* 𝔤	4 ft. long.	16 cyl. ft.
	D	Diameter of cylinder.	1	D	2 ft. diam.	1

The solidity thus obtained is merely the product of the three dimensions which become multiplied together into one product by the above operations of the rule; consequently those products will express the solidity in various terms, either cubic, or cylindrical feet; cubic or cylindrical inches; square inch feet, or cylindrical inch feet, &c. according to the different terms in which the given dimensions of the solid are expressed. But by dividing these different expressions of the solidities, by suitable numbers, they may be converted into any other required measures of solidities.

The necessary division of the product may be effected in the same operation of the rule, by taking some other number upon the line D, instead of 1, as is directed in the above precepts. The number to be taken for that purpose is called a gauge point, and it must be the square root of the required divisor, as will be more fully explained in another article; it is only intended here to show how the rule at one operation, multiplies the three dimensions of the solid together into one product, and also divides that product by any required divisor.

To find how many times the area of one square, or circle, is greater than that of another square or circle. This is best done by the lines C and D, thus.

Find the side of the largest square (or the diameter of largest circle) upon D, and set the slider so that 1 upon C corresponds therewith; then find the side of the smallest square (or the diameter of the smallest circle) upon D, and opposite to it on C will be the number of times that the area of the smaller is contained in that of the larger.

Sliding Rule.	C	1	Proportion of areas.	*Exam.* C	1	4 times.
	D	Diam. of large circle	Diam. of small circle.	D	3 ft. dia.	6 ft. dia.

To find the corresponding diameters and lengths, of a number of square prisms or cylinders which will have the same solidity. This can be done by the lines 𝔤 and D, with the slider inverted thus.

The side of any square prism, or the diameter of any cylinder, being found upon D, the slider must be set so that the length of that prism or cylinder upon 𝔤 corresponds therewith; the rule then forms a table, which shows the lengths on 𝔤, and on D the sides of square prisms, or diameters of cylinders, corresponding to those lengths, so as to exhibit the dimensions of a number of square prisms or cylinders having the same solidity.

Sliding Rule, slide inverted.	𝔤	Length of cylinder.	Length of cylinder.	*Exam.* 𝔤	5 ft. long.	1·25 ft. long.
	D	Diam. of cylinder.	Diam. of cylinder.	D	2 ft. diam.	4 ft. diam.

If the solidity of these cylinders is required, it may be found at the same set of the rule, upon the line 𝔤 opposite to some particular point upon the line D, which is called a gauge point; and there are different gauge points for different cases, as will be explained.

In cases when the dimensions of the cylinder are given in feet, and the solidity is required in cubic feet, the gauge point will be at 113 or 357 upon D; by this means we may find the dimensions of all kinds of cylinders, which will contain a given number of cubic feet, the slider being set so that the given number of cubic feet on line 𝔤, corresponds with 113 or 357 on C, thus.

Sliding Rule, slide inverted.				*Exa.*			
	Ↄ	Solidity cubic ft.	Lengths of cylinders ft.		Ↄ	15·71 cub. ft.	5 ft. long.
	D	113 or 357.	Diam. of cylinders in ft.		D	113 or 357.	2 ft. dia.

In cases of square prisms, when all the dimensions are given in feet, and the solidity in cubic feet, then the gauge point on D will be 1 or 10. If the sides of the square prisms are required in inches, and their lengths in feet, the solidity being given in cubic feet; then the gauge point will be 12 or 379 upon D. Or in cases of cylinders, when their diameters are required in inches, and their lengths in feet, the solidity being given in cubic feet; the gauge point will be 135 or 423 on D. The nature of these gauge points will be further explained in the proper place.

TO FIND THE AREAS OF CIRCLES. The areas and circumferences of circles are so frequently required by engineers, that it is very desirable to have tables ready calculated to show them by inspection(*a*); but when a table is not at hand, the sliding rule is a good substitute; it may be set either for a particular case, or it may be set so as to form a table.

To find the area of a circle in square inches, having given its diameter in inches.

RULE. Multiply the square of the diameter in inches, by the decimal ·785398; the product is the area in square inches. *Note*, dividing the square by 1·273239, which is the reciprocal of the decimal number, will give the same result as multiplying by the decimal number.

Example. 25 inches diameter, squared = 625 circular inches area × ·7854 = 490 87 square inches. Or 625 circular inches ÷ 1·273 = 490·87 square inches.

Sliding Rule, slide inverted.				*Exam.*			
	A	Diam. of circle inches.	1·273		A	25 inc. dia.	1·273
	Ↄ	Diam. of circle inches.	Area squ. inc.		Ↄ	25 inc. dia.	491 squ. inc.

This requires the rule to be set for each operation, but the lines C and D being placed as follows, will form a complete table of the areas of circles in square inches.

Sliding Rule.				*Exam.*				
	C	95	Area square inches.		95	49 squ. inc.	or	77
	D	11	Diameter in inches.		11	7·9 inches.		9·9

(*a*) The author felt the want of a table of the areas of circles, at the commencement of his professional studies, and being unable to find such a table in any book, he was induced to calculate the areas and circumferences of circles, from one inch in diameter to one hundred, with every intermediate half inch, to form a table, which he has had in use for twenty years past.

A more extensive table, containing every quarter of an inch of diameter, has since been calculated by the late Mr. Goodwyn, the proprietor of an extensive brewery in London. This gentleman had always shown a great spirit for improvements, he was the first person who adopted Mr Watt's rotative steam-engine in 1784 (see p. 434); and after his retirement from business, he devoted much time to researches into the properties of numbers, and methods of computation, some of which were published during his life. The table in question has been printed by Dr. Gregory, in his Mathematics for Practical Men, 8vo., 1825. During the progress of the present work, the author has also found a table of the areas of circles for every inch in diameter, in Sir Samuel Morland's Elevation des Eaux, 1685, (see p. 92), and the same table is printed in Ozanam's Cours de Mathematique, 1697, Vol. III.

The author's table has been verified from both these sources, so that every inch in diameter, in the above table, is the result of three independent calculations; and every half inch is derived from two independent calculations; the intermediate quarters of inches are Mr. Goodwyn's calculations; and the author has proved as many of them as could be done by comparison with each other. It has been thought sufficient, for all practical purposes, to give the numbers to five places of figures, but they have been calculated to seven places of decimals, and the last figure of decimals in the table is the nearest which could be chosen to the truth. The foundation of the calculations in this table are as follows.

The circumference of any circle is 3·14159265 times its diameter. Or, reciprocally, the diameter of any circle is ·31830989 of its circumference.

The area of any circle is ·78539816 of the area of its circumscribing square. Or, reciprocally, the area of any square is 1·2732395 times the area of the circle which may be inscribed within it.

A square foot contains 144 square inches; or (144 × 1·2732=) 183·346 circular inches.

TABLE OF THE CIRCUMFERENCES AND AREAS OF CIRCLES OF ALL DIAMETERS, FROM ONE QUARTER OF AN INCH TO ONE HUNDRED INCHES.

The diameters are expressed in inches and quarters; the circumferences in inches; and the areas in circular inches, in square inches, and in square feet.

Circumf.	Area.	Diam.	Area.		Circumf.	Area.	Diam.	Area.	
Inches.	Circular inc.	Inches.	Square inc.	Square feet.	Inches.	Circular inc.	Inches.	Square inc.	Square feet.
·7854	·0625	0·25	·04909	·00034	38·484	150·06	12·25	117·86	·81847
1·5708	·25	0·5	·19635	·00136	39·270	156·25	12·5	122·72	·85221
2 3562	·5625	0·75	·44179	·00307	40·055	162 56	12·75	127·68	·88664
3·1416	1·	1·	·7854	·00545	40·841	169·	13·	132·73	·92175
3·927	1·5625	1·25	1·2272	·00852	41·626	175·56	13·25	137·89	·95754
4·7124	2·25	1·5	1·7671	·01227	42·411	182·25	13·5	143·14	·99402
5·4978	3·0625	1·75	2·4053	·01670	43 197	189·06	13·75	148·49	1·0312
6·2832	4·	2·	3·1416	·02182	43·982	196·	14·	153·94	1·0690
7 0686	5 0625	2·25	3·9761	·02761	44·768	203·06	14·25	159·48	1·1075
7·854	6·25	2·5	4·9087	·03409	45·553	210·25	14 5	165·13	1·1467
8 6394	7·5625	2·75	5·9396	·04125	46·338	217·56	14·75	170·87	1·1866
9·4248	9·	3·	7·0686	·04909	47·124	225·	15·	176·71	1·2272
10·210	10·562	3·25	8·2958	·05761	47 909	232·56	15·25	182·65	1·2684
10·996	12 25	3·5	9·6211	·06681	48 695	240·25	15 5	188·69	1 3104
11 781	14·062	3·75	11·045	·07670	49·480	248·06	15·75	194·83	1·3530
12·566	16·	4·	12·566	·08727	50·265	256·	16·	201·06	1·3963
13·352	18·062	4·25	14·186	09851	51·051	264·06	16·25	207·39	1·4402
14·137	20·25	4·5	15·904	·11045	51·836	272·25	16·5	213·82	1·4849
14 923	22·562	4·75	17·721	·12306	52·622	280·56	16·75	220·35	1·5302
15·708	25·	5·	19·635	·13635	53 407	289	17·	226·98	1 5762
16·493	27·562	5·25	21 648	·15033	54·192	297·56	17·25	233·71	1·6230
17·279	30·25	5·5	23·758	·16499	54 978	306·25	17·5	240·53	1·6703
18·064	33·062	5·75	25 967	·18033	55·763	315·06	17·75	247·45	1·7184
18·85	36·	6·	28.274	·19635	56·549	324·	18·	254·47	1·7671
19·635	39·062	6·25	30·680	·21305	57·334	333.06	18·25	261·59	1·8166
20·420	42·25	6·5	33·183	·23044	58·119	342·25	18 5	268·80	1·8667
21·206	45·562	6·75	35·785	·24850	58 905	351·56	18·75	276·12	1·9175
21 991	49·	7·	38·485	·26725	59·690	361·	19·	283·53	1·9689
22·777	52·562	7·25	41·282	·28668	60·476	370·56	19 25	291·04	2·0211
23·562	56·25	7·5	44·179	·30680	61·261	380·25	19·5	298·65	2·0740
24·347	60·062	7·75	47·173	·32759	62·046	390·06	19·75	306·35	2·1275
25·133	64·	8·	50·265	·34907	62·832	400·	20·	314·16	2·1817
25·918	68·062	8·25	53·456	·37122	63·617	410·06	20·25	322·06	2 2366
26·704	72 25	8·5	56·745	·39406	64·403	420 25	20 5	330·06	2·2921
27·489	76·562	8 75	60·132	·41758	65·188	430·56	20·75	338·16	2 3484
28·274	81·	9·	63·617	·44179	65·973	441·	21·	346 36	2·4053
29·060	85·562	9·25	67·201	·46667	66·759	451·56	21·25	354·66	2 4629
29 845	90·25	9·5	70·882	·49224	67·544	462 25	21·5	363 05	2·5212
30·631	95 062	9·75	74 662	·51849	68 33	473 06	21 75	371·54	2 5802
31·416	100·	10·	78 540	·54542	69·115	484·	22·	380·13	2·6398
32·201	105·06	10·25	82·516	57303	69·900	495·06	22 25	388·82	2·7001
32 987	110·25	10·5	86·590	·60132	70·686	506·25	22·5	397·61	2·7612
33·772	115 56	10·75	90 762	63029	71·471	517·56	22·75	406·49	2·8229
34 558	121·	11·	95·033	·65995	72·257	529·	23	415·48	2 8852
35 343	126·56	11·25	99·402	·69029	73·042	540·56	23·25	424·56	2 9483
36 128	132 25	11·5	103 87	·72131	73·827	552·25	23·5	433·74	3·0121
36·914	138·06	11·75	108·43	·75301	74 613	564·06	23·75	443·01	3 0765
37 699	144·	12·	113 10	·78540	75 398	576	24·	452 39	3 1416

Circumf.	Area.	Diam.	Area.		Circumf.	Area.	Diam.	Area.	
Inches.	Circular inc.	Inches.	Square inc.	Square feet.	Inches.	Circular inc.	Inches.	Square inc.	Square feet.
76·184	588·06	24·25	461·86	3·2074	113·88	1314·06	36·25	1032·06	7·1671
76·969	600·25	24·5	471·44	3·2739	114·67	1332·25	36·5	1046·35	7·2663
77·754	612·56	24·75	481·11	3·3410	115·45	1350·56	36·75	1060·73	7·3662
78·540	625·	25·	490·87	3·4088	116·24	1369·	37·	1075·21	7·4667
79·325	637·56	25·25	500·74	3·4774	117·02	1387·56	37·25	1089·79	7·5680
80·111	650·25	25·5	510·71	3·5466	117·81	1406·25	37·5	1104·47	7·6699
80·896	663·06	25·75	520·77	3·6164	118·60	1425·06	37·75	1119·24	7·7725
81·681	676·	26·	530·93	3·6870	119·38	1444·	38·	1134·11	7·8758
82·467	689·06	26·25	541·19	3·7582	120·17	1463·06	38·25	1149·09	7·9798
83·252	702·25	26·5	551·55	3·8302	120·95	1482·25	38·5	1164·16	8·0844
84·038	715·56	26·75	562·00	3·9028	121·74	1501·56	38·75	1179·32	8·1897
84·823	729·	27·	572·56	3·9761	122·52	1521·	39·	1194·59	8·2958
85·608	742·56	27·25	583·21	4·0501	123·31	1540·56	39·25	1209·95	8·4025
86·394	756·25	27·5	593·96	4·1247	124·09	1560·25	39·5	1225·42	8·5098
87·179	770·06	27·75	604·81	4·2000	124·88	1580·06	39·75	1240·98	8·6179
87·964	784·	28·	615·75	4·2761	125·66	1600·	40·	1256·64	8·7266
88·750	798·06	28·25	626·80	4·3528	126·45	1620·06	40·25	1272·39	8·8361
89·535	812·25	28·5	637·94	4·4301	127·23	1640·25	40·5	1288·25	8·9462
90·321	826·56	28·75	649·18	4·5082	128·02	1660·56	40·75	1304·20	9·0569
91·106	841·	29·	660·52	4·5870	128·81	1681·	41·	1320·25	9·1684
91·892	855·56	29·25	671·96	4·6664	129·59	1701·56	41·25	1336·40	9·2806
92·677	870·25	29·5	683·49	4·7465	130·38	1722·25	41·5	1352·65	9·3934
93·462	885·06	29·75	695·13	4·8273	131·16	1743·06	41·75	1369·00	9·5069
94·248	900·	30·	706·86	4·9087	131·95	1764·	42·	1385·44	9·6211
95·033	915·06	30·25	718·69	4·9909	132·73	1785·06	42·25	1401·98	9·7360
95·819	930·25	30·5	730·62	5·0737	133·52	1806·25	42·5	1418·63	9·8516
96·604	945·56	30·75	742·64	5·1572	134·30	1827·56	42·75	1435·36	9·9678
97·389	961·	31·	754·77	5·2414	135·08	1849·	43·	1452·20	10·0847
98·175	976·56	31·25	766·99	5·3263	135·87	1870·56	43·25	1469·14	10·2023
98·960	992·25	31·5	779·31	5·4119	136·66	1892·25	43·5	1486·17	10·3206
99·746	1008·06	31·75	791·73	5·4981	137·44	1914·06	43·75	1503·30	10·4396
100·531	1024·	32·	804·25	5·5851	138·23	1936·	44·	1520·53	10·5592
101·316	1040·06	32·25	816·86	5·6727	139·02	1958·06	44·25	1537·86	10·6796
102·102	1056·25	32·5	829·58	5·7609	139·80	1980·25	44·5	1556·28	10·8075
102·887	1072·56	32·75	842·39	5·8499	140·59	2002·56	44·75	1572·81	10·9223
103·673	1089·	33·	855·30	5·9396	141·37	2025·	45·	1590·43	11·0447
104·458	1105·56	33·25	868·31	6·0299	142·16	2047·56	45·25	1608·15	11·1677
105·243	1122·25	33·5	881·41	6·1209	142·94	2070·25	45·5	1625·97	11·2914
106·029	1139·06	33·75	894·62	6·2126	143·73	2093·06	45·75	1643·89	11·4159
106·814	1156·	34·	907·92	6·3050	144·51	2116·	46·	1661·90	11·5409
107·600	1173·06	34·25	921·32	6·3981	145·30	2139·06	46·25	1680·02	11·6667
108·385	1190·25	34·5	934·82	6·4918	146·08	2162·25	46·5	1698·23	11·7932
109·170	1207·56	34·75	948·42	6·5862	146·87	2185·56	46·75	1716·54	11·9204
109·956	1225·	35·	962·11	6·6813	147·65	2209·	47·	1734·94	12·0482
110·741	1242·56	35·25	975·91	6·7771	148·44	2232·56	47·25	1753·45	12·1767
111·527	1260·25	35·5	989·80	6·8736	149·23	2256·25	47·5	1772·05	12·3059
112·312	1278·06	35·75	1003·79	6·9707	150·01	2280·06	47·75	1790·76	12·4358
113·097	1296·	36·	1017·88	7·0686	150·80	2304·	48·	1809·56	12·5664

To find the area of a circle in square feet; having given, its diameter in inches.

RULE. Divide the square of the diameter in inches, by 183·346. The quotient is the area in square feet.

Example. 25 inc. diameter squared = 625 circular inches area, ÷ 183·346 = 3·4088 square feet.

Sliding Rule, slide inverted.	A Diam. of circle inches.	183·34	*Example.*	A	25 inc. dia.	183·34
	Ɔ Diam. of circle inches.	Area square feet.		Ɔ	25 inc. dia.	3·409

Or the lines C and D will form a complete table of diameters and areas when the slider is set thus:

Sliding Rule.	C 3·14	Area square feet.	*Example.*	C 3·14	15 squ. ft.	or	22
	D 24	Diameter inches.		D 24	52·5 inc.		63·5.

Circumf.	Area.	Diam.	Area.		Circumf.	Area.	Diam.	Area.	
Inches.	Circular inc.	Inches.	Square inc.	Square feet.	Inches.	Circular inc.	Inches.	Square inc.	Square feet.
151·58	2328·06	48·25	1828·46	12·698	189·28	3630·06	60·25	2851·04	19·799
152·37	2352·25	48·5	1847·45	12·829	190·07	3660·25	60·5	2874·75	19·964
153·15	2376·56	48·75	1866·55	12·962	190·85	3690·56	60·75	2898·56	20·129
153·94	2401·	49·	1885·74	13·095	191·64	3721·	61·	2922·47	20·295
154·72	2425·56	49·25	1905·83	13·235	192·42	3751·56	61·25	2946·47	20·462
155·51	2450·25	49·5	1924·42	13·364	193·21	3782·25	61·5	2970·57	20·629
156·29	2475·06	49·75	1943·91	13·499	193·99	3813·06	61·75	2994·77	20·797
157·08	2500·	50·	1963·49	13·635	194·78	3844·	62·	3019·07	20·966
157·96	2525·06	50·25	1983·18	13·772	195·56	3875·06	62·25	3043·47	21·135
158·65	2550·25	50·5	2002·96	13·909	196·35	3906·25	62·5	3067·96	21·305
159·44	2575·56	50·75	2022·84	14·047	197·13	3937·56	62·75	3092·55	21·476
160·22	2601·	51·	2042·82	14·186	197·92	3969·	63·	3117·25	21·648
161·01	2626·56	51·25	2062·90	14·326	198·71	4000·56	63·25	3142·03	21·820
161·79	2652·25	51·5	2083·07	14·466	199·49	4032·25	63·5	3166·92	21·993
162·58	2678·06	51·75	2103·34	14·607	200·28	4064·06	63·75	3191·91	22·166
163·36	2704·	52·	2123·72	14·748	201·06	4096·	64·	3216·99	22·340
164·15	2730·06	52·25	2144·19	14·890	201·85	4128·06	64·25	3242·17	22·515
164·93	2756·25	52·5	2164·75	15·033	202·63	4160·25	64·5	3267·45	22·691
165·72	2782·56	52·75	2185·42	15·176	203·42	4192·56	64·75	3292·83	22·867
166·50	2809·	53·	2206·18	15·321	204·20	4225·	65·	3318·31	23·044
167·29	2835·56	53·25	2227·04	15·466	204·99	4257·56	65·25	3343·88	23·221
168·07	2362·25	53·5	2248·01	15·611	205·77	4290·25	65·5	3369·55	23·400
168·86	2889·06	53·75	2269·06	15·757	206·56	4323·06	65·75	3395·33	23·579
169·65	2916·	54·	2290·22	15·904	207·35	4356·	66·	3421·19	23·758
170·43	2943·06	54·25	2311·48	16·052	208·13	4389·06	66·25	3447·16	23·939
171·22	2970·25	54·5	2332·83	16·200	208·92	4422·25	66·5	3473·23	24·120
172·00	2997·56	54·75	2354·28	16·349	209·70	4455·56	66·75	3499·39	24·301
172·79	3025·	55·	2375·83	16·499	210·49	4489·	67·	3525·65	24·484
173·57	3052·56	55·25	2397·48	16·649	211·27	4522·56	67·25	3552·01	24·667
174·36	3080·25	55·5	2419·22	16·800	212·06	4556·25	67·5	3578·47	24·850
175·14	3108·06	55·75	2441·07	16·952	212·84	4590·06	67·75	3605·03	25·035
175·93	3136·	56·	2463·01	17·104	213·63	4624·	68·	3631·68	25·220
176·71	3164·06	56·25	2485·05	17·257	214·41	4658·06	68·25	3658·43	25·406
177·50	3192·25	56·5	2507·19	17·411	215·20	4692·25	68·5	3685·28	25·592
178·29	3220·56	56·75	2520·42	17·503	215·98	4726·56	68·75	3712·23	25·779
179·07	3249·	57·	2551·76	17·721	216·77	4761·	69·	3739·28	25·967
179·86	3277·56	57·25	2574·19	17·876	217·56	4795·56	69·25	3766·43	26·156
180·64	3306·25	57·5	2596·72	18·033	218·34	4830·25	69·5	3793·67	26·345
181·43	3335·06	57·75	2619·35	18·190	219·13	4865·06	69·75	3821·01	26·535
182·21	3364·	58·	2642·08	18·348	219·91	4900·	70·	3848·45	26·725
183·00	3393·06	58·25	2664·90	18·506	220·70	4935·06	70·25	3875·99	26·917
183·78	3422·25	58·5	2687·83	18·665	221·48	4970·25	70·5	3903·63	27·109
184·57	3451·56	58·75	2710·85	18·825	222·27	5005·56	70·75	3931·36	27·301
185·35	3481·	59·	2733·97	18·986	223·05	5041·	71·	3959·19	27·494
186·14	3510·56	59·25	2757·19	19·147	223·84	5076·56	71·25	3987·12	27·688
186·92	3540·25	59·5	2780·51	19·309	224·62	5112·25	71·5	4015·15	27·883
187·71	3570·06	59·75	2803·92	19·472	225·41	5148·06	71·75	4043·28	28·078
188·50	3600·	60	2827·43	19·635	226·19	5184·	72·	4071·50	28·274

To find the side of a square which will have the same area as a circle of a given diameter.

RULE. Multiply the diameter of the given circle by the constant decimal ·8862269. The product will be the side of the equivalent square.

Sliding rule. { A 39 Side of equivalent square. / B 44 Diameter of the circle. } *Exam.* A 39 square 70. / B 44 circle 79 diam.

The rule being thus set, the lines A and B will form a table showing the diameters of all circles, and the sides of their equivalent squares. Or it may be done by the lines C and D, thus:

Sliding rule. { C 70 55 / D Diam. of circle. Side of equivalent square. } *Exam.* C 70 55. / D circle 79 diam. square 70.

Circumf.	Area.	Diam.	Area.		Circumf.	Area.	Diam.	Area.	
Inches.	Circular inc.	Inches.	Square inc.	Square feet.	Inches.	Circular inc.	Inches.	Square inc.	Square feet.
226·98	5220·06	72·25	4099·83	28·471	270·96	7439·06	86·25	5842·63	40·574
227·76	5256·25	72·5	4128·25	28·668	271·75	7482·25	86·5	5876·55	40·809
228·55	5292·56	72·75	4156·77	28·866	272·53	7525·56	86·75	5910·56	41·046
229·34	5329·	73·	4185·39	29·065	273·32	7569·	87·	5944·68	41·282
230·12	5365·56	73·25	4214·10	29·265	274·10	7612·56	87·25	5978·89	41·520
230·91	5402·25	73·5	4242·92	29·465	274·89	7656·25	87·5	6013·20	41·758
231·69	5439·06	73·75	4271·83	29·665	275·67	7700·06	87·75	6047·61	41·997
232·48	5476·	74·	4300·84	29·867	276·46	7744·	88·	6082·12	42·237
233·26	5513·06	74·25	4329·95	30·069	277·25	7788·06	88·25	6116·73	42·477
234·05	5550·25	74·5	4359·16	30·272	278·03	7832·25	88·5	6151·43	42·718
234·83	5587·56	74·75	4388·46	30·475	278·82	7876·56	88·75	6186·24	42·960
235·62	5625·	75·	4417·86	30·680	279·60	7921·	89·	6221·14	43·202
236·40	5662·56	75·25	4447·37	30·884	280·39	7965·56	89·25	6256·14	43·445
237·19	5700·25	75·5	4476·97	31·090	281·17	8010·25	89·5	6291·24	43·689
237·98	5738·06	75·75	4506·66	31·296	281·96	8055·06	89·75	6326·43	43·934
238·76	5776·	76·	4536·46	31·503	282·74	8100·	90·	6361·73	44·179
239·55	5814·06	76·25	4566·35	31·711	283·53	8145·06	90·25	6397·12	44·424
240·33	5852·25	76·5	4596·35	31·919	284·31	8190·25	90·5	6432·61	44·671
241·12	5890·56	76·75	4626·44	32·128	285·10	8235·56	90·75	6468·20	44·918
241·90	5929·	77·	4656·63	32·338	285·88	8281·	91·	6503·88	45·166
242·69	5967·56	77·25	4686·91	32·548	286·67	8326·56	91·25	6539·67	45·414
243·47	6006·25	77·5	4717·30	32·759	287·46	8372·25	91·5	6575·55	45·664
244·26	6045·06	77·75	4747·78	32·971	288·24	8418·06	91·75	6611·53	45·913
245·04	6084·	78·	4778·36	33·183	289·03	8464·	92·	6647·61	46·164
245·83	6123·06	78·25	4809·04	33·396	289·81	8510·06	92·25	6683·79	46·415
246·62	6162·25	78·5	4839·82	33·610	290·60	8556·25	92·5	6720·06	46·667
247·40	6201·56	78·75	4870·70	33·825	291·38	8602·56	92·75	6756·44	46·920
248·19	6241·	79·	4901·67	34·039	292·17	8649·	93·	6792·91	47·173
248·97	6280·56	79·25	4932·74	34·255	292·95	8695·56	93·25	6829·48	47·427
249·76	6320·25	79·5	4963·91	34·472	293·74	8742·25	93·5	6866·15	47·682
250·34	6360·06	79·75	4995·18	34·689	294·52	8789·06	93·75	6902·91	47·937
251·33	6400·	80·	5026·55	34·907	295·31	8836·	94·	6939·78	48·193
252·11	6440·06	80·25	5058·01	35·125	296·10	8883·06	94·25	6976·74	48·449
252·90	6480·25	80·5	5089·58	35·344	296·88	8930·25	94·5	7013·80	48·707
253·68	6520·56	80·75	5121·24	35·564	297·67	8977·56	94·75	7050·96	48·965
254·47	6561·	81·	5153·00	35·785	298·45	9025·	95·	7088·22	49·224
255·25	6601·56	81·25	5184·85	36·006	299·24	9072·56	95·25	7125·58	49·483
256·04	6642·25	81·5	5216·81	36·228	300·02	9120·25	95·5	7163·03	49·743
256·83	6683·06	81·75	5248·86	36·450	300·81	9168·06	95·75	7200·58	50·004
257·61	6724·	82·	5281·02	36·674	301·59	9216·	96·	7238·23	50·265
258·40	6765·06	82·25	5313·27	36·898	302·38	9264·06	96·25	7275·98	50·528
259·18	6806·25	82·5	5345·62	37·122	303·16	9312·25	96·5	7313·82	50·790
259·97	6847·56	82·75	5378·06	37·348	303·95	9360·56	96·75	7351·77	51·054
260·75	6889·	83·	5410·61	37·574	304·73	9409·	97·	7389·81	51·318
261·54	6930·56	83·25	5443·25	37·800	305·52	9457·56	97·25	7427·95	51·583
262·32	6972·25	83·5	5475·99	38·028	306·31	9506·25	97·5	7466·19	51·849
263·11	7014·06	83·75	5508·82	38·256	307·09	9555·06	97·75	7504·53	52·115
263·89	7056·	84·	5541·77	38·485	307·88	9604·	98·	7542·96	52·382
264·68	7098·06	84·25	5574·81	38·714	308·66	9653·06	98·25	7581·50	52·649
265·46	7140·25	84·5	5607·94	38·944	309·45	9702·25	98·5	7620·13	52·917
266·25	7182·56	84·75	5641·17	39·175	310·23	9751·56	98·75	7658·86	53·186
267·04	7225·	85·	5674·50	39·406	311·02	9801·	99·	7697·69	53·456
267·82	7267·56	85·25	5707·93	39·638	311·80	9850·56	99·25	7736·61	53·726
268·61	7310·25	85·5	5741·46	39·871	312·59	9900·25	99·5	7775·64	53·997
269·39	7353·06	85·75	5775·08	40·105	313·37	9950·06	99·75	7814·76	54·269
270·18	7396·	86·	5808·80	40·339	314·16	10000·	100·	7853·98	54·542

TABLE OF DIVISORS FOR CALCULATING THE QUANTITIES OF MATTER IN SQUARE PRISMS, CYLINDERS, OR SPHERES, BY THE SLIDING RULE.

Mr. Watt and Mr. Southern calculated a series of numbers to form a concise table, which is engraved at the back of the Soho rule, to serve as theorems for the mensuration of solid bodies. The solidities of regular bodies will be represented by the products which are obtained by multiplying three of their principal dimensions together, viz., length, breadth, and thickness; and then, by dividing those products by suitable divisors, the quotients will express the solidities in any terms that may be required (see p. 550).

For instance, suppose a cylinder to be 2 feet diameter, and 5 feet long; its capacity will be (2 ft. diam. squared = 4 circular feet area, × 5 ft. long =) 20 cylindrical feet. If it is required to express that capacity in cubic feet, we must divide the 20 cylindrical feet by 1·2732, (because one cubic foot is equal to that number of cylindrical feet,) and we have 15·708 cubic feet. Or, instead of dividing, multiplying by the decimal number ·7854 (which is the reciprocal of 1·2732) will give the same result in cubic feet; because one cylindrical foot is equal to that fraction of a square foot.

By means of the lines of squares and roots, C and D, on the sliding rule, we can, at one operation, multiply the square of a number by any other number, and then divide the product by a third number, so as to obtain the quotient without any necessity for observing or recording, either the square or the intermediate product. This property has been already explained, p. 550, and it is a great convenience in saving time, and avoiding errors.

To perform the calculation, all the four lines of the Soho sliding rule are used at once, thus; The proper divisor being found on the line A, the slider must be set, so that the length of the cylinder on B, will correspond with that divisor on A; then the diameter of the cylinder being found on the line D, the contents of the cylinder will stand opposite to it on the line C.

Soho Sliding Rule, all four lines.		*Example.*	
A	Divisor 1·273	A	Divisor 1·273
B	Length of cylinder feet.	B	Length 5 feet.
C	Content of cylinder cubic feet.	C	15·708 cubic feet.
D	Diameter of cylinder feet.	D	2 feet diameter.

The rule being thus set, it forms a complete table of the contents of all cylinders which are of the same length, but of different diameters; for opposite to any diameter on D, the cubic contents will be found on C. Or the rule may be set as follows, and then it will form a complete table of the contents of all cylinders which are of the same diameter, but of different lengths, for opposite to any length on the line B, will be the content on the line A.

Soho Sliding Rule, all four lines.		*Example.*	
A	Content of cylinder cubic feet.	A	15·708 cubic feet.
B	Length of cylinder in feet.	B	5 feet long.
C	Divisor 1·273	C	Divisor 1·273
D	Diameter of cylinder feet.	D	Diameter 2 feet.

The operation of the divisors being now explained, we may proceed to state how they are arranged in the table which is engraved at the back of the rule. The different titles, Cubic feet, Cubic inches, Water pounds, Cast iron lbs., &c., along the top of the table, denote the terms in which the contents of solids will be expressed, when they are calculated by means of the divisors which are arranged under each title.

The different columns marked FF, FI, and II, under each title, denote the particular divisors which are to be used in each case, according to the terms in which the dimensions of the solids are given, as the data for the calculation. Thus FF denotes that all the dimensions are given in feet; FI, that the length is given in feet, and the diameter in inches; and II denotes that all the dimensions are given in inches.

Table of Divisors for calculating the solidities or weights of square prisms, cylinders, or globes.

Solids.	Solidity in cubic feet.			Solidity in cubic inches.		Water pounds, a cubic foot 62·5 lbs.			Cast iron lbs. a cubic foot 450 lbs.			Wrought iron lbs. a cubic foot 485 lbs.		
	FF	FI	II	FI	II	FF	FI	II	FF	FI	II	FF	FI	II
Squares . .	1·	144	1728	·08333	1·	·016	2·304	27·648	·00222	·32	3·84	·002062	·2969	3·563
Cylinders	1·273	183·3	2200	·1061	1·273	·02037	2·933	35·203	·00283	·4074	4·889	·002625	·378	4·536
Globes . . .	1·91	—	3300	—	1·91	·03056	—	52·804	·00424	—	7·334	·003938	—	6·805
						Specific gravity 1.			Specific grav. 7·2			Specific grav. 7·76		

Solids.	Lead lbs. a cubic foot 710 lbs.			Copper lbs. a cubic foot 555 lbs.			Brass lbs. a cubic foot 525 lbs.			Stone lbs. a cubic foot 155 lbs.			Brick lbs. a cubic foot 125 lbs.		
	FF	FI	II	FF	FI	II	FF	FI	II	FF	FI	II	FF	FI	II
Squares . .	·00141	·2028	2 434	·00180	·2595	3·114	·0019	·2743	3·291	·00645	·929	11·148	·008	1·152	13·824
Cylinders.	·00179	.2582	3·099	·00229	·3304	3·964	·00243	·3492	4·191	·00821	1·182	14·195	·01019	1·466	17·601
Globes . . .	.00269	—	4·648	·00344	—	5·946	·00364	—	6·286	·00123	—	21·292	·01528	—	26·402
	Spec.grav. 11·36			Spec. grav. 8·88			Specific grav. 8·4			Specific grav. 2·48			Specific gravity 2.		

The three different lines of the table are marked at the beginning with the words Squares, Cylinders, and Globes, to denote what kind of solid each divisor is applicable to. The principal divisors in the above table are obtained in the following manner (*a*):

The three divisors in the column marked FF, under the title of cubic feet, are to be used in cases when all the dimensions of the solids are given in feet, and their solidities are required in cubic feet. The product which is obtained in such cases by multiplying the three dimensions together, will represent the solidities of the different solids in different terms, viz., in cubic feet, if the solid is a square prism; or in cylindrical feet, if it is a cylinder; or in spherical feet, if it is a globe. The several divisors must be adapted to reduce all those different denominations to cubic feet; and they are the number of cubic feet, cylindrical feet, or spherical feet, which are equal to one cubic foot. Thus, The divisor for square prisms is 1, because no reduction is required. The divisor for cylinders is 1·27324, for that number of cylindrical feet are contained in one cubic foot. The divisor for spheres is 1·90986, for that number of spherical feet make one cubic foot.

The three divisors in the column marked II, under the title of Cubic inches, are the same numbers as the above, being the number of cubic inches, cylindrical inches, or spherical inches, which are equal to one cubic inch.

The two divisors marked FI, under the title of Cubic feet, are to be used for square prisms or cylinders, when their lengths being given in feet, and their other dimensions in inches, their solidities are required in cubic feet. In such cases the products which are obtained by multiplying the three dimensions together, will express the solidities in different terms, viz. The solidities of square prisms will be expressed in square inch feet, that is, square prisms one inch square and one foot long. And the solidities of cylinders, in cylindrical inch feet, that is, cylinders one inch diameter, and one foot long. The divisors are 144, which is the number of square inch feet in one cubic foot. And 183·346, which is the number of cylindrical inch feet in one cubic foot.

The three divisors marked II under the title of Cubic feet, are to be used when all the dimensions are given in inches, and the solidities are required in cubic feet. The product of the multiplication of the three dimensions will then give the solidities in cubic inches, cylindrical inches, or spherical inches. The divisor for square prisms is 1728 cubic inches in a cubic foot. For cylinders 2200·15 cylindrical inches in a cubic foot. And for globes 3300·23 spherical inches in a cubic foot.

The two dvisors marked FI, under the title Cubic inches, are to be used when the lengths are given in feet, and the other dimensions in inches, and the solidities are required in cubic inches. The divisor for square prisms is ·08333, because that decimal portion of a square inch foot, is equal to one

(*a*) The above table of divisors was calculated by the author, and it is more complete and exact than the table usually engraved at the back of the sliding rule.

cubic inch. And the divisor for cylinders is ·1061, for that portion of a cylindrical inch foot is equal to one cubic inch.

The divisors for cylinders are 1·27324 times the corresponding divisors for square prisms, under the same title and denomination; and the divisors for globes are 1·90986 times the corresponding divisors for square prisms. Or the divisors for globes are 1·5 times the corresponding divisors for cylinders. The divisors which are marked FI under each title, are 144 times the corresponding divisor marked FF under the same title; and the divisors II are 12 times the corresponding divisors FI; consequently those marked II are 1728 times those marked FF.

The divisors which are placed under the different titles of Water, Cast iron, &c. are to find the weight, in pounds avoirdupois, of different solids composed of those substances. Those divisors which are marked FF, are the number of cubic feet, cylindrical feet, or spherical feet of each substance, that will weigh one pound. The divisors marked FI are the number of square inch feet, or cylindrical inch feet of each substance, that will weigh one pound. And the divisors marked II, are the number of cubic inches, cylindrical inches, or spherical inches, of each substance, that will weigh one pound.

For instance, a cubic foot of water weighs 62·5 pounds; and under the title Water, the divisors marked FF are as follows: ·016 of a cubic foot of water weighs one pound (that is, a pound of water is = ·016 of a cubic foot); ·02037 of a cylindrical foot weighs one pound; and ·03056 of a spherical foot weighs one pound. The divisors marked FI are as follows; 2·304 square inch feet of water weigh one pound; or 2·9335 cylindrical inch feet of water weigh one pound. The divisors marked II are, 27·648 cubic inches of water weigh one pound; or 35·2025 cylindrical inches of water weigh one pound; or 52·8038 spherical inches of water weigh one pound.

The divisors for calculating the weight of any other substance in pounds, may be obtained by dividing the proper divisors for water by the specific gravity of the substance in question. Or else by dividing the proper divisors for cubic feet, or cubic inches, by the weight in pounds, of a cubic foot, or a cubic inch, of that substance.

For instance, the specific gravity of cast iron is 7·2; that is, any bulk of cast iron is 7·2 times the weight of an equal bulk of water. And ·016 of a cubic foot of water weigh one pound; therefore (·016 ÷ 7·2 =) ·00222 of a cubic foot of cast iron will weigh one pound; and that number is the proper divisor for finding the weight of square prisms of cast iron in pounds, when all the dimensions are given in feet: accordingly it is marked in the table. Cast iron lbs. FF, Squares ·00222.

In like manner the divisor cast iron lbs. FI cylinders is (2·9335 ÷ 7·2 =) ·40743. This division may be performed by the sliding rule, with the slide inverted, thus:

Sliding rule,	A	1	Divisor for that sub.	*Ex.*	A	1	·407 divis.
slide inverted.	Ɔ	Divis. for wat. lbs.	Specific grav. of sub.		Ɔ	2·93 wat. FI	7·2 sp. gr.

When the weight of a cubic foot, or of a cubic inch, of any substance is given in pounds, divisors for calculating the weights of solids of that substance may be found by dividing the divisors under the titles of cubic feet or cubic inches, by the weight in pounds of a cubic foot, or a cubic inch of that substance. For instance, the weight of a cubic foot of cast iron is (7·2 × 62·5 =) 450 pounds. The divisor for cubic feet, cylinder, FI, is 183·346 which ÷ 450 gives ·40743, which is the proper divisor for cast iron cylinders, FI, as before. Or by the sliding rule with the slide inverted.

Sliding rule,	A	1	Divisor for that substance.	*Ex.*	A	1	·407
slide inverted.	Ɔ	Divisor for cubic feet.	Weight of a cubic foot, lbs.		Ɔ	183·3	450lbs.

To explain the mode of finding divisors for new cases, we may suppose that divisors are wanted to calculate what weight of water in tons (of 2240 pounds) will be contained in different vessels, all the dimensions of those vessels being given in feet. The products obtained by the multiplications of the three dimensions, will express the solidities in cubic feet, or cylindrical feet, or spherical feet, and the divisors must therefore be the number of cubic feet, cylindrical feet, or spherical feet, which will weigh one ton, or 2240 pounds. As a cubic foot of water weighs 62·5 pounds, we have (2240 lbs. ÷ 62·5 lbs. =) 35·84 cubic feet of water weigh a ton, for the divisor for square prisms. Or a cylindrical foot of water weighs 49·087 pounds; therefore (2240 lbs. ÷ 49·087 lbs. =) 45·633 cylindrical feet of water weigh one ton: this is the divisor for cylinders. And a spherical foot of water weighs 32·725 pounds; hence (2240 lbs. ÷ 32·725 lbs. =) 68·449 spherical feet of water weigh a ton: this is the divisor for globes.

It is obvious that the divisors which will give the weight of bodies in tons, must be 2240 times those divisors which will give their weight in pounds; or for hundred weights 112 times, &c. Hence to obtain divisors for calculating the weights of bodies in tons, or in hundred weights, we have only to multiply the numbers in the table by 2240 lbs. or by 112 lbs. *Example.* The divisor for the weight of cylinders of water FF in pounds, is ·016; this multiplied by 2240 lbs. is = 35·84 for the divisor for the weight of cylinders of water, FF, in tons, as before.

Examples of the use of the table of divisors. A cylindrical piston-rod of wrought iron being $3\frac{1}{2}$ inches diameter and 9 feet long, how many pounds will it weigh? The divisor for this case is ·378 according to the table; for the solid being a cylinder, the required divisor must be in the middle line, of the three horizontal lines of the table; and it will be under the head of Wrought iron, in the column FI, because the length of the cylinder is given in feet, whilst its diameter is in inches. The divisor may be taken on the line A thus, and the rule will form a table showing the weight of every different diameter, when the length is 9 feet.

Soho Sliding Rule, all four lines.	A	·378 divisor.		(without altering the slider.)		
	B	9 feet long.				
	C	292 pounds weight.	or	214 pounds.	or	251 pounds.
	D	3·5 inches diameter.		3 inc. diam.		3·25 inc. diam.

Or the divisor may be taken on the line C thus, and the rule will form a table, showing the weight corresponding to every different length, the diameter being always $3\frac{1}{2}$ inches.

Soho Sliding Rule, all four lines.	A	292 pounds weight.	or	324 pounds.	or	259 pounds.
	B	9 feet long.		10 feet long.		8 feet long.
	C	·378 divisor.		(without altering the slider.)		
	D	3·5 inches diameter.				

Gage points for the sliding rule. If the square root of any divisor is taken upon the line D, it will point out the same result upon the line C or g, as the divisor itself does upon the lines A or C or g. The square roots of divisors are called gage points; for instance, the square root of 1·273 is 1·128, which is the gage point corresponding to the divisor 1·273, and may be used thus on the line D.

Sliding Rule.	C Length of cylinder ft.	Content of cylinder cubic ft.	*Ex.*	C	5 ft. long.	15·708 cub. ft.
	D Gage point 1·13 or 3·57.	Diameter of cylinder in feet.		D	1·13 g. p.	2 feet diam.

When the rule is thus set, it forms a table for all cylinders which are of different diameters, but of the same length; for opposite to any diameter on D, the content will be found on C.

If the slider is inverted and set as follows, then the rule will form a table of the corresponding lengths and diameters of a number of cylinders, which will have the same content that is pointed out by the gage point; for opposite to any diameter on D will be the requisite length on g.

Sliding Rule, slide inverted.	g Content of cylinder cub.ft.	Length of cylinder in ft.	*Ex.*	g	15·708 cub.ft.	5 ft.long.
	D Gage point 1·13 or 3·57	Diameter of cylinder ft.		D	1·13 g. p.	2 ft.diam.

For instance, in the above example, a cylinder 1 foot diameter must be 20 feet long, in order to contain 15·708 cubic feet; or a cylinder 3 feet diameter would require to be 2·22 feet long.

If it is required to find the product which results from the multiplication of the three dimensions of the solid, it may be found on the line g, opposite to 1 on the line D; in the above case, that product is the solidity of the cylinder expressed in cylindrical feet.

Note. Instead of taking the square root of the divisor for a gage point on the line D, we may take the square root of ten times that divisor, and it will give the same results; this will be apparent by trial with the rule; the square root of 12·73 is 3·568, and opposite to that number on the line D, we shall find 15·708 on C or g, which is the same result as is opposite to the other gage point 1·128; because the series of numbers on the line C or g, are twice repeated. (See table, p. 569.)

To calculate the solidities of rectangular prisms, such as planks of wood, or flat bars of metal, by the sliding rule with the table of divisors. In these cases we must first calculate the proper size for a square prism, whereof the sectional area would be the same as that of the rectangular prism in question, and then the calculation of its solidity or weight may be made, in the same manner as if it were a square prism. The two sides of the rectangle must be multiplied together to represent the sectional area by their product, and the square root of that product will be the side of a square which will have the same area as the rectangle.

For instance, a flat bar 2 inches thick, by $4\frac{1}{2}$ broad $(2 \times 4\frac{1}{2}) = 9$ square inches, the square root of which is 3 inches, and a square bar of that size would be equivalent to the flat bar. This may be done by the sliding rule thus:

Sliding rule, slide inverted.	A	Large side of rectangle.	Side of equivalent square.	This is where the same numbers on both lines correspond.
	D	Small side of rectangle.	Side of equivalent square.	

Or still more conveniently by the lines C and D. The number representing one of the sides of the rectangle (either the largest or the smallest) being found upon one of the lines C or D, the slider must be placed with that number corresponding to the same number upon the other line. Then the number representing the other side, being found upon the line C, the side of the equivalent square will be opposite to it, upon the line D, thus:

Sliding rule.	C	Large side of rectan.	Small side of rectangle.	*Ex.*	C	$4\frac{1}{2}$ broad	2 thick.
	D	Large side of rectan.	Side of equivalent square.		D	$4\frac{1}{2}$ broad	3 square.

By the same process we may find the diameter of a circle which shall have the same area as that of any given ellipsis; whereby an elliptical prism may be assimilated to a cylinder thus:

Sliding rule.	C	Conjugate dia. of ellips.	Transverse diameter.	*Ex.*	C	16 con. dia.	9 trans. dia.
	D	Conjugate dia. of ellips.	Diam. of equal circle.		D	16 con. dia.	12 dia. of cir.

To calculate the solidities of pyramids or cones by the sliding rule, with the table of divisors. The solidity of any pyramid, or cone, is one-third of that of a prism, or cylinder, of the same base and vertical height as the pyramid, or cone; hence we must calculate the solidity, or weight, of a prism, or cylinder, of the same base and vertical height as the pyramid, or cone, by the sliding rule, with the proper divisor, or gage point, taken from the table; and then one-third of the result, will be the solidity, or weight, of the pyramid, or cone.

The solidities of similar pyramids, or cones, are as the cubes of the sides, or diameters, of their bases respectively; similar pyramids, or cones, are those which have the same angle at the vertex, and therefore the dimensions of their bases, bear some constant proportion to their vertical heights.

To find the solidities of frustums of square pyramids, or cones; having given, the vertical height of the frustum, its diameter at the base, and its diameter at the top; all the dimensions being in the same terms, either feet or inches, &c.

RULE. Divide the difference between the cubes of the sides, or diameters, of the two ends, by the difference between those sides, or diameters; and multiply the quotient by one third of the vertical height of the frustum. The product is the solidity, which will be expressed in cubic feet, or cubic inches, if it is the frustum of a square prism; or in cylindrical feet, or cylindrical inches, if it is a conical frustum.

Note. The square root of the quotient, which is obtained by dividing the difference between the cubes of the sides, or diameters, by the difference between those sides, or diameters, will be the diameter of a square prism, or of a cylinder, whereof the height being equal to that of the frustum, it will have three times the solidity: the following rule is deduced from this fact.

To find the diameter of a cylinder which will have the same solidity as that of a given frustum of a pyramid or cone.

Rule. Divide the difference between the cubes of the sides, or diameters, by three times the difference between the sides, or diameters, and then extract the square root of the quotient. That root will be the diameter of a cylinder, which, being of the same height as the frustum, will have the same solidity. The solidity or weight of such equivalent prism, or cylinder, may then be calculated with the sliding rule, by means of a suitable divisor, or gage point, selected from the tables.

Example. Suppose a conical frustum to be 6 feet vertical height; 5 feet diameter at the base; and 3 feet diameter at the top. The cube of 5 is 125; from which deduct 27 (the cube of 3), and the remainder is 98, for the difference between the cubes; divide this by (three times 2, the difference between 5 ft. diam. and 3 ft. diam. =) 6, and the quotient is $16\frac{1}{3}$; the square root of this is 4·04 feet for the diameter of a cylinder, which being 6 feet high, will have the same content as the frustum in question. Thus 4·04 ft. dia. squared = $16\frac{1}{3}$ circular feet area, $\times$ 6 ft. high = 98 cylindrical feet, or $\times$ ·7854 = 76·96 cubic feet. If it had been a frustum of a square prism, the content would have been 98 cubic feet.

This calculation will require three operations by the sliding rule; first, two operations to find the cube of the side, or diameter, of the base, and the cube of the side, or diameter, of the top, thus:

Sliding rule,	g	Side or diameter.	Cube of side or diam.	*Ex.* g	5 ft. dia.	125 cube.
slide inverted.	D	Side or diameter.	1	D	5 ft. dia.	1

The difference between the two cubes cannot be taken by the sliding rule; but it must be found by subtracting the cube of the greatest side, or diameter, from the cube of the smallest side or diameter (thus, 125 — 27 = 98); and also the difference between those sides, or diameters, must be taken; (thus, 5 ft. dia. — 3 ft. dia. = 2). We may then find the side of an equivalent square prism, or the diameter of an equivalent cylinder, of the same height as the frustum, thus:

Sliding rule,	g	Dif. of cubes.	3 times dif. of dia.	*Ex.* g	98 dif. cub.	$(3 \times 2 =)$ 6.
slide inverted.	D	1	Diam. of cylinder.	D	1	4·04 diam.

Note. As the same numbers are twice repeated on the line g, there are two different results which may be obtained in each case, by following the above precept; and care must be taken to choose the right one, which may be known by its being rather greater than a mean between the two diameters of the frustum. For instance, in the above example, one 6 on the line g will point out 4·04 ft. diam.; and the other 6 will point out 1·28 ft. diam.; so that there is no danger of mistaking them in this case, if we are aware of the fact that there are two different results.

In some cases of frustums it is desirable to know what would be the solidity of the whole pyramid or cone, if it were completed; and then the solidity of the upper part, or small pyramid, which is wantiug, being also calculated, the difference between the two solidities will be the solidity of the frustum.

To find the vertical height of the complete pyramid, or cone, having given, the vertical height of the frustum, the side or diameter of its base, and the side or diameter of its top.

Rule. Multiply the vertical height of the frustum, by the side, or diameter of its base; and divide the product by the difference between the side, or diameter, of the base, and the side, or diameter, of the top. The quotient will be the whole height of the complete pyramid, or cone.

Example. Suppose a conical frustum to be 6 feet vertical height, 5 feet diameter at the base, and 3 feet diameter at the top. Then, 5 feet diameter at base, $\times$ 6 feet high = 30 $\div$ by (5 ft. — 3 ft. =) 2 feet difference of diameters = 15 feet, would be the height of the complete cone.

Sliding Rule,	A	Diameter of base.	Height of complete cone.	*Ex.* A	5 ft. diam.	15 ft. high.
slide inverted.	Ↄ	Height of frustum.	Difference of diameters.	Ↄ	6 ft. high.	2 ft. diff.

The solidity of a cone 5 feet diameter at the base, and 15 feet high, is as follows: 5 feet diameter squared = 25 circular feet area, × (one-third of 15 feet high =) 5 = 125 cylindrical feet; or, if it had been a square prism, the solidity would have been 125 cubic feet. The small cone which is wanting, is 3 feet diameter at the base, and 9 feet high. Therefore, 3 squared = 9 × (one-third of 9 ft. high =) 3 = 27 cylindrical feet. Hence, the solidity of the frustum must be 125 — 27 = 98 cylindrical feet; or 98 cubic feet if it had been a square prism.

To find the solidity of an ellipsoid, or spheroid. The solidity of an ellipsoid is two-thirds of that of its circumscribing cylinder; hence, we may calculate the solidity or weight of such a cylinder by the sliding rule, with the aid of a suitable divisor or gage point from the table; and then take two-thirds of the result, for the solidity or weight of the ellipsoid in question.

As the divisors for globes are $1\frac{1}{2}$ times those for cylinders, they will enable us to calculate the true result for ellipsoids, as well as for spheres, at one operation. The length of the axis of the ellipsoid is to be taken on the line B or C, and the diameter of the great or equatorial circle of the ellipsoid on the line D. Thus,

Sliding Rule, all four lines.	A	Divisor for a globe.	*Exam.*	A	Divis. for cub. ft. FF. 1·91
	B	Length of axis of ellipsoid.		B	9 ft. length of axis,
	C	Solidity or weight of ellipsoid.		C	230 cubic feet.
	D	Equatorial diam. of ellipsoid.		D	7 feet diameter.

Or, if the square root of the divisor is taken as a gage point on the line D, instead of the divisor itself upon A, then the rule may be set thus, (See the table of gage points, p. 569.)

Sliding Rule, slide inverted.	g	Solidity or weight of ellipsoid.	Length of axis.	*Ex.*	g	230 cb. ft.	9 ft. lon.
	D	Gage point for a globe.	Diam. of equator.		D	138 g. p.	7 ft. dia.

To find the cube of any given number by the lines C *and* D, *on the sliding rule.*

Sliding Rule.	C	Number to be cubed.	Cube of the number.	*Ex.*	C	4 numb.	64 cube.
	D	1	Number to be cubed.		D	1	4 numb.

Or thus, by the lines g and D when the slide is inverted, which is the best method.

Sliding Rule, slide inverted.	g	Number to be cubed.	Cube of the number.	*Ex.*	g	4 numb.	64 cube.
	D	Number to be cubed.	1		D	4 numb.	1

To extract the cube root of any number by the lines g and D on the sliding rule, with the slide inverted. This is done according to the last precept, with the slider inverted, by finding the number whose root is to be extracted, upon the line g, and placing it opposite to 1 on the line D; then seeking along the lines g and D, for the place where the divisions representing the same numbers on both lines, meet together, those numbers are the cube root required. Thus,

Sliding Rule, slide inverted.	g	Numb.	Cube root.	This is where the same numbers on both lines correspond.	*Exa.*	g	64 cube.	4 root.
	D	1	Cube root.			D	1	4 root.

Note. If two divisions which are of the same value, upon the lines g and D, do not exactly coincide, the coincident point will be within the space between those two divisions of the same value, which are nearest to a coincidence. There are three such points of coincidence of similar numbers along the lines, one denoting the cube root required; the others the cube roots of 10 times, and of 100 times the number; care must be taken to choose the proper root of the three, but there is nothing on the rule to point it out.

The following precepts show how the sliding rule can perform successive multiplications, and divisions, of numbers with the squares, and cubes, of other numbers, by one operation of the lines C and D; or by all the four lines of the Soho rule. These precepts will be very useful guides to those calculators who require to adapt the sliding rule to new cases. Two precepts are given for each case, to show the process which must be gone through, either with the slider direct, or with the slider inverted; and the calculator can select that which is most suitable to his particular purpose.

To square a given number, and multiply that square by another given number, at one operation. For instance, multiply the square of 6 by 4.

The square root of the multiplicand is 6; the square of which is (6 × 6 =) 36 for the multiplicand itself. The multiplier is 4. And 144 is the resulting product.

Sliding Rule.	C	Multiplier	Resulting product.	*Ex.*	C	4 mult.	144 result.
	D	1	Sq. root of multiplicand.		D	1	6 sq. rt.
Sliding Rule,	ꓭ	Multiplier.	Resulting product.	*Ex.*	ꓭ	4 mult.	144 result.
slide inverted.	D	Sq. root of multiplicand.	1		D	6 sq. rt.	1

To square a given number, and divide that square by another given number, at one operation. For instance, divide the square of 12 by 6.

The square root of the dividend is 12; the square of which is (12 × 12 =) 144 for the dividend itself. The divisor is 6. And 24 is the resulting quotient.

Sliding Rule.	C	Resulting quotient.	Divisor.	*Ex.*	C	24 result.	6 divis.
	D	Square root of dividend.	Divisor.		D	12 sq. rt.	6 divis.
Sliding Rule,	ꓭ	Divisor.	Resulting quotient.	*Ex.*	ꓭ	6 divis.	24 result.
slide inverted.	D	Square root of dividend.	Divisor.		D	12 sq. rt.	6 divis.

To cube a given number, and divide that cube by the square of another given number, at one operation. For instance, divide the cube of 8 by the square of 4.

The cube root of the dividend is 8; the cube of which is (8 × 8 × 8 =) 512 for the dividend itself. The square root of the divisor is 4; the square of which is (4 × 4 =) 16 for the divisor itself. And 32 is the resulting quotient.

Sliding Rule.	C	Cube root of dividend.	Resulting quotient.	*Ex.*	C	8 cube rt.	32 result.
	D	Square root of divisor.	Cube root of dividend.		D	4 sq. root.	8 cube rt.
Sliding Rule,	ꓭ	Cube root of dividend.	Resulting quotient.	*Ex.*	ꓭ	8 cube rt.	32 result.
slide inverted.	D	Cube root of dividend.	Sq. root of divisor.		D	8 cube rt.	4 sq. root.

To multiply the square of a given number, by another given number, and divide the product by the square of a third given number, at one operation. For instance, multiply the square of 8 by 12, and divide the product by the square of 4.

The square root of the multiplicand is 8, the square of which is (8 × 8 =) 64, the multiplicand itself. The multiplier is 12. And 768 is the product, which, being divided twice successively by the square root of the divisor (thus, 768 ÷ 4 = 192 ÷ 4 =) gives 48 for the resulting quotient.

Sliding Rule.	C	Multiplier.	Resulting quotient.	*Ex.*	C	12 mult.	48 result.
	D	Sq. root of divisor.	Sq. root of multiplicand.		D	4 sq. rt. div.	8 sq. rt. mult.
Sliding Rule,	ꓭ	Multiplier.	Result. quotient.	*Ex.*	ꓭ	12 mult.	48 result.
slide inverted.	D	Sq. root of multiplicand.	Sq. root of divisor.		D	8 sq. rt. mult.	4 sq. rt. div.

Note. This is the same operation as that by which the solidities, or weights, of square prisms, or cylinders, or spheres, are calculated by the aid of gage points; which are the square roots of the divisors given in the table at the back of the Soho rule. See p. 559 and 569.

To cube a number, and divide that cube by another given number, at one operation. For instance, divide the cube of 8 by 16.

The cube root of the dividend is 8; the cube of which is 512 for the dividend itself. The divisor is 16. And 32 is the resulting quotient. This operation cannot be performed by the lines C and D alone, but it requires all the four lines of the Soho rule, thus,

Soho Rule, all four lines.	A	Divisor.	*Example.*	A	16 divisor.
	B	Cube root of dividend.		B	8 cube root.
	C	Resulting quotient.		C	32 result.
	D	Cube root of dividend.		D	8 cube root.

Soho Rule, all four lines, slide inverted.	A	Divisor.	*Example.*	A	16 divisor.
	Ɔ	Resulting quotient.		Ɔ	32 result.
	ᗺ	Cube root of dividend.		ᗺ	8 cube root.
	D	Cube root of dividend.		D	8 cube root.

To multiply the square of a given number by another given number, and divide the product by a third given number; at one operation. For instance, multiply the square of 8 by 16, and divide the product by 8.

The square root of the multiplicand is 8; the square of which is (8 × 8 =) 64 for the multiplicand itself. The multiplier is 16. And 1024 is the product; which, divided by the divisor 8, gives 128 for the resulting quotient. This operation requires all the four lines of the Soho rule.

Soho Rule, all four lines.	A	Divisor.	*Example.*	A	8 divisor.
	B	Multiplier.		B	16 multiplier.
	C	Resulting quotient.		C	128 resulting quotient.
	D	Square root of multiplicand.		D	8 sq. root of multiplicand.

Soho Rule, slide inverted, all four lines.	A	Divisor.	*Example.*	A	8 divisor.
	Ɔ	Resulting quotient.		Ɔ	128 resulting quotient.
	ᗺ	Multiplier.		ᗺ	16 multiplier.
	D	Square root of multiplicand.		D	8 sq. root of multiplicand.

Note. This is the same operation as that by which the solidities, or weights, of square prisms or cylinders, are calculated by means of the divisors in the table p. 557, at the back of the Soho rule.

To multiply two given numbers together, and divide their product by the square of a third given number, at one operation. For instance, multiply 16 by 9, and divide the product by the square of 6.

The multiplicand is 16. And 9 is the multiplier. Their product is 144; which divided twice successively by 6 the square root of the divisor (thus, 144 ÷ 6 = 24 ÷ 6 =), gives 4 for the resulting quotient. This operation requires all the four lines of the Soho rule.

Soho Rule, all four lines.	A	Multiplicand.	*Example.*	A	16 multiplicand.
	B	Resulting quotient.		B	4 result.
	C	Multiplier.		C	9 multiplier.
	D	Square root of divisor.		D	6 sq. root of divisor.

Soho Rule, slide inverted, all four lines.	A	Multiplicand.	*Example.*	A	16 multiplicand.
	Ɔ	Multiplier.		Ɔ	9 multiplier.
	ᗺ	Resulting quotient.		ᗺ	4 result.
	D	Square root of divisor.		D	6 sq. root of divisor.

To divide a given number by the square of another given number, at one operation. For instance, divide 288 by the square of 6.

The dividend is 288; which divided twice successively by 6, the square root of the divisor, (thus, $288 \div 6 = 48 \div 6 =$) gives 8, for the resulting quotient.

Sliding rule.	C	Dividend.	Resulting quotient.	*Exa.*	C	288 div.	8, result.
	D	Square root of divisor.	1		D	6 sq. rt.	1
Sliding rule, slide inverted.	Ↄ	Dividend.	Resulting quotient.	*Exa.*	Ↄ	288 div.	8, result.
	D	1	Square root of divisor.		D	1	6 sq. root.

To find such an unknown number as will bear the same proportion to a given number, as that which exists between the squares of two other given numbers. For instance, as the square of 4, is to the square of 6: so is 12, to a fourth unknown number. The terms of the ratio are $(4 \times 4) = 16$, and $(6 \times 6 =)$ 36; it is an increasing proportion, and therefore the smallest of those terms (16) must be used for the divisor of the product which is obtained by the multiplication of the largest term 36, by the known number 12.

Thus, $\frac{36}{16}$ths of $12 = 27$ is the fourth unknown number; for 12 the known number $\times 36 = 432 \div 16 = 27$. The arithmetical process is in reality, $12 \times 6 = 72 \times 6 = 432$ product, which, $\div 4 = 108$ for the first quotient, and $\div 4 = 27$, resulting quotient, which is the fourth unknown number.

Sliding rule.	C	Known number.	Unknown number.	*Ex.*	C	known 12.	unknown 27.
	D	Sq. rt. small. term.	Sq. rt. large term.		D	sq. rt. 4.	sq. rt. 6.
Sliding rule, slide inverted.	Ↄ	Unknown number.	Known number.	*Ex.*	Ↄ	unknown 27.	known 12.
	D	Sq. rt. small term.	Sq. rt. large term.		D	sq. rt. 4.	sq. rt. 6.

To divide a number by the square root of another number at one operation. For instance, divide 18 by the square root of 9.

The dividend is 18. The square of the divisor is 9; the square root of which is 3, for the real divisor. The resulting quotient is $(18 \div 3 =)$ 6.

Sliding Rule.	C	Square of divisor.	1	*Exa.*	C	9 sq. of divis.	1
	D	Dividend.	Result. quotient.		D	18 dividend.	6 result.
Sliding Rule, slide inverted.	Ↄ	1	Square of divisor.	*Exa.*	Ↄ	1	9 sq. of divis.
	D	Dividend.	Result. quotient.		D	18 divid.	6 result.

To extract the square root of a number, and multiply that root by another number. For instance, extract the square root of 16, and multiply that root by 7.

The square of the multiplicand is 16; the square root of which is 4, for the real multiplicand. The multiplier is 7. The resulting product is $(4 \times 7 =)$ 28.

Sliding Rule.	C	1	Sq. of multiplicand.	*Exa.*	C	1	16 sq. of mult.
	D	Multiplier.	Resulting product.		D	7 mult.	28 result.
Sliding Rule, slide inverted.	Ↄ	1	Sq. of multiplicand.	*Exa.*	Ↄ	1	16 sq. of mult.
	D	Resulting prod.	Multiplier.		D	28 result.	7 multiplier.

To divide the square root of one number, by the square root of another number. For instance, divide the square root of 64, by the square root of 16.

The square of the dividend is 64; the square root of which is 8, for the real dividend. The square of the divisor is 16, the square root of which is 4, for the real divisor. The resulting quotient is $(8 \div 4 =)$ 2.

Sliding Rule.	C	Squ. of dividend.	1	*Exa.*	Ɔ	64 sq. of divid.	1
	D	Squ. of divisor.	Result. quotient.		D	16 sq. of divis.	2 result.

Sliding Rule, slide inverted.	Ɔ	Squ. of dividend.	Sq. of divisor.	*Exa.*	Ɔ	64 sq. of divid.	16 sq. of divis.
	D	1	Result. quotient.		D	1	2 result.

Precepts of this kind might be greatly extended, but the above will serve for those operations which are of the most common occurrence. When a long calculation has been made by a series of arithmetical operations in the usual manner, the sliding rule by the aid of one or other of the above precepts, will, in most cases, be found capable of performing several of the succeeding operations at once, so as to abridge the process very considerably.

To find the hypothenuse of a right-angled triangle, by one operation with the sliding rule; having given the dimensions of the base, and of the perpendicular, both in the same terms, as feet or inches, &c. For instance, suppose the base to be 4 feet, and the perpendicular 3 feet, what will be the hypothenuse in feet?

The usual arithmetical process is to add the square of the base, to the square of the perpendicular; viz. $(4 \times 4 =)16 + (3 \times 3 =)9 = 25$; and then the square root of their sum being extracted; viz. 5, that root is the hypothenuse required.

The same result may be obtained thus: Divide (9) the square of the perpendicular, by (16) the square of the base; the quotient will be a decimal fraction, (·5625), to which prefix 1·, then multiply (1·5625) the number so obtained, by (16) the square of the base; the product will be the square of the hypothenuse; and the square root of that product, viz. (5), will be the hypothenuse required.

The latter process, though it appears complicated in figures, can be very readily performed by the lines C and D of the sliding rule, with the slider direct, thus: Find the base on the line D, and set the slider so that 1 upon the line C, corresponds to that base; then opposite to the perpendicular on D, is the required decimal fraction, to which 1 is to be prefixed. The number formed by that addition being sought upon C, the required hypothenuse will be found opposite to it on D, thus,

Sliding rule.	C	1	Decim. fract.	&	1· + Dec. frac.	*Ex.*	C	1	·5625 dec.	&	1. 5625
	D	Base.	Perpendicular.		Hypothenuse.		D	4 base.	3 perpend.		5 hypoth.

Note. This is one of the theorems invented by Mr. Watt, or Mr. Southern.

Directions to Engineers for the choice of a Sliding Rule.

The slider of a sliding rule should fit very accurately into its groove, and must slide very freely in it, without being so loose as to drop, or move by its own weight. The face of the slider should form a very even surface with the face of the rule, when it is put into the groove either way, direct or reversed. Sliding rules are commonly made of box wood; but short ones of 11 or 13 inches should be made of very white ivory, and highly polished. In all cases the rules should be made a long time before the divisions are engraved upon them, that the wood or ivory may become seasoned, and shrink as much as it is disposed to do. The larger rules, from 18 to 30 inches, can only be made of box wood; and the very long ones of lance wood. It is of great importance to have a light-coloured surface for the ground of the divisions, as they can be read so much more easily, and with less fatigue to the eye. No substance is so good as ivory for this reason, but on account of the expense of that substance, and the small demand for excellent sliding rules, the instrument makers do not construct them of ivory, except when expressly ordered.

The goodness of the divisions of sliding rules must depend upon the accuracy of the original patterns, and the skill of the engraver. The rules made by Mr. Bate are more correctly divided than any others that the author has examined.

The accuracy of the principal divisions may be tried by comparing the coincidences of the divisions representing numbers which are multiples of each other, and if these coincidences are examined with the slider in several positions, both when direct and when inverted, the errors will discover themselves if they are of a sensible magnitude. The regularity of the intermediate subdivisions which fill up between the primary divisions, may be judged of by the eye with considerable accuracy.

Engineers are usually provided with a Soho sliding rule, and they should also have an inverted sliding rule; these have hitherto been made as separate instruments, which is not so convenient for use, as if they were combined into one. Persons who are already accustomed to use the Soho rule, and who have acquired facility in calculating by it, will wish to retain an instrument to which they are habituated; but the author has found the advantages of the inverted rule to be so great in his own practice, that he most strongly recommends it to the notice of engineers, and, consequently, in forming the precepts which are given in this work, a preference has been given to the slide inverted, except in cases where there is some good reason for using the slide direct. The advantages of the inverted method can only be attained in part, when the slide of the Soho rule is inverted, because the divisions are only half the size that they would be, upon a proper inverted rule of the same length; and therefore such an inverted rule should always be used when the precepts are entitled slide inverted, and have the letters A and Ɔ prefixed.

A very convenient sliding rule for the use of engineers who are accustomed to the Soho rule, may be made with two sliders, one in the back, and the other in the front face of the rule; one face being engraved to form a complete Soho rule, such as is represented in the sketch, p. 537, and the other face being engraved with an inverted broken line upon the slider, as is represented in the sketch, p. 541. The latter should be used in all cases which are directed to be performed with the slide inverted, and the lines A and Ɔ. The divisions on the rule will be the same at both sides. The table of divisors may be engraved at the back of the extra slider, and in the bottom of its groove.

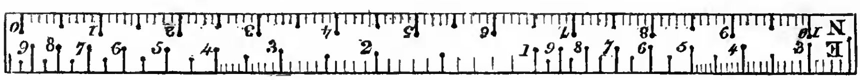
This mode of combining the Soho rule with the inverted rule, the author has found to answer the intended purpose very well; but such a rule has a double quantity of divisions, and he afterwards found that the same advantages may be attained, by combining four lines upon a rule with one slider, leaving all the back of the rule to receive tables of gage points, and specific gravities. This new arrangement he recommends as that which will be found the most convenient for engineers, who have not already become familiar with the use of the Soho rule; for it has no more divisions upon it than the Soho rule, and, with a very few exceptions, it will perform all the calculations for which precepts are given in this work; and it has some new properties.

A new arrangement of Logarithmic Lines upon a Sliding Rule, by the Author.

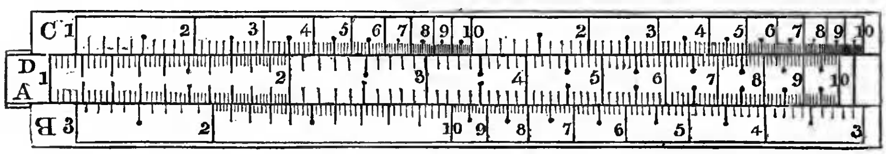
The inverted slide rules which have been hitherto made, contain only two lines of single radius; viz. one placed direct on the rule, and the other placed inverted on the slider, and broken into two parts, as already described, p. 541. This rule is very convenient for performing multiplication and division, and all cases of proportion or rule of three, the advantages of certainty in reading off the results, and of accuracy, from the large size of its divisions, have been already stated.

For calculations which involve the square roots, or the squares of numbers, it is requisite to have a line of double radius acting against a line of single radius; but the slide rules which are usually constructed (such as the Soho rule) require three lines of double radius and one line of single radius, which occupy the entire face of the rule, and leave no room for an inverted line, without having another additional slider, with an inverted line to fit the groove of the Soho rule, as has been already described.

To avoid the inconvenience of changing the slider, and to make a complete sliding rule for engineers, with only one slider, the author has made the following arrangement of four logarithmic lines; (*a*)



{ *back of the slider.*



viz., three of single radius and one of double radius, which are so disposed upon one face of a sliding rule, as to perform multiplication, division, and simple proportion, on the inverted method, by means of the line of single radius A, at the lower part of the slider, acting against the inverted broken line ꓭ, at the lower part of the rule. And all cases of squares, or square roots, may be solved by means of the line of single radius D, at the upper part of the slider, acting against the line of double radius C, at the upper part of the rule.

By this means the properties of the inverted rule, and also those of the Soho rule, are attained by the new sliding rule, with only one slider, and with no more divisions than the Soho rule has upon it; and three of the lines being single radius instead of double radius, their divisions are twice as large, on a rule of a given length, so as to be more exact, and less fatiguing to the eye to read off.

This new rule is composed of the two lower lines of the Soho rule, disposed over the two lines of the inverted rule, and the only change made in the action of either of them is, that those lines which were formerly engraved upon the rule, are now put upon the slider, and *vice versa*. The two upper lines of the Soho rule are omitted, because their place is more advantageously supplied by the inverted lines at the lower part of the new rule.

The divisors in the table which is engraved on the back of the Soho rule, for calculating the contents, or the weights of prisms, cylinders, or spheres, of different substances, are adapted to be used on the upper line A of that rule (see p. 556), and are not applicable to this new rule; but the following table of gage points is engraved on the back of the new rule, to show the gage points which must be used on the line D for such calculations. The numbers in this table are the square roots of the numbers in the former table of divisors (p. 559), or the square roots of ten times these divisors, as is explained at p. 560.

(*a*) Mr. Bate has undertaken to make sliding rules of this kind, according to the author's directions, with very accurate divisions; he expects the new rules will be found more suitable for the use of engineers than any similar instruments which are now to be procured.

Table of Gage Points for calculating the solidities or weights of square prisms, cylinders, or globes, by the sliding rule.

SOLIDS.	Cubic Feet.			Cubic Inch.		Water, lbs.			Tons.		Cast Iron, lbs.		
Dimensions.	FF	FI	II	FI	II	FF	FI	II	FF	FI	FF	FI	II
Squares	1	12	416	289	1	4	48	166	599	72	149	179	62
	1	379	131	913	1	126	152	526	189	227	471	566	196
Cylinders	113	135	469	103	113	45	54	188	214	811	168	202	70
	357	428	148	326	357	143	171	593	675	256	532	638	221
Globes	138	—	574	—	138	175	—	23	262	993	206	—	271
	437	—	182	—	437	553	—	727	827	314	652	—	856

SOLIDS.	Bar Irn. lbs.		Brass, lbs.		Lead, lbs.		Copper, lbs.		Stones, lbs.		Bricks, lbs.	
Dimensions.	FI	II	FI	II	FI	II	FI	II	FF	F I	FF	FI
Squares	172	189	166	181	45	156	161	56	254	305	283	339
	545	597	524	574	142	493	509	176	803	964	894	107
Cylinders	194	213	187	205	51	176	182	63	287	344	319	383
	615	673	591	647	161	557	575	199	906	109	101	121
Globes	—	261	—	251	—	216	—	77	111	—	391	—
	—	825	—	793	—	682	—	244	351	—	124	—

The gage points in the above table are to be taken upon the line D of the sliding rule, in the manner stated in page 559. The lengths of square prisms, cylinders, globes, or ellipsoids, are to be taken upon the line C, if the slider is direct, or on ꓘ if it is inverted; and their sides or diameters upon the line D. Their solidities or weights will be found upon the line C, if the slider is direct, or on ꓘ if it is inverted.

If the slider of the rule is direct, then the length upon C must be set opposite to either of the gage points upon D: and the solidity or weight will be found upon C, opposite to the side or the diameter upon D; thus,

Sliding rule.				*Ex.*			
	C	Length of cylinder.	Solidity or weight.		C	210 ft long.	660 cub. feet.
	D	Gage points.	Diam. of cylinder.		D	135 or 428	24 inc. diam.

If the slider is inverted, then the length upon ꓘ must be set opposite to the diameter upon D: and the solidity or weight will be found upon ꓘ, opposite to either of the gage points upon D; thus,

Sliding rule, slide inverted.				*Ex.*			
	ꓘ	Length of cylinder.	Solidity or weight.		ꓘ	210 ft. long.	660 cub. ft.
	D	Diam. of cylinder.	Gage points.		D	24 inc. diam.	135 or 428.

The above table contains two gage points for each case, but as either of them will give the same result, they may be taken indifferently upon the line D; the number which stands at top is to be preferred for common use, as being the most convenient and exact. In the table which is engraved at the back of the new sliding rule, only those upper numbers are inserted, and those which stand beneath are omitted on account of room; but on the long rules the complete table may be engraved as above.

In addition to the above table of gage points, a table of specific gravities is engraved at the back of the sliding rule, for the purpose of calculating the weight of solids of substances for which no gage points are given. Having found by means of the gage points for water, what would be the weight of the solid in water, the result may be multiplied by the specific gravity of the substance in question. Or else we may calculate the quantity of matter in cubic feet, and then multiply the result by the weight of a cubic foot of the substance in question.

A Table of the Specific Gravities of different Substances, and the weight of a cubic foot in pounds; adapted to be engraved at the back of a sliding rule.

Platina	21·04	710	Lead	11·36	Glass	2·88	163	Purbeck	2·61	Ivory	1·92
Gold	19·36	555	Copper	8·88	Marble	2·72	160	Portland	2·56	Horn	1·84
Mercury	13·55	525	Brass	8·40	Slate	2·67	155	Millstone	2·48	Bone	1.65
Silver	10·51	485	Iron	7·76	Granite	2·64	152	Paving	2·43	Box	1·33
Steel	7·84	450	Cast Iron	7·20	China	2·38	125	Brick	2·00	Coal	1·28

60	Wax	·96	44	Ash	·70	Carbonic acid gas	544	Times lighter than water. Barom. 30 inches. Thermom. 60 degr.
58	Oil	·93	40	Fir	·64	Oxygen gas .	747	
55	Oak	·88	37	Elm	·59	Azotic gas .	854	
47	Beech	·75	35	Deal	·56	Common air .	830	
45	Mahogany	·72	15	Cork	·24	Hydrogen gas	11960	

The divisors and gage points in the preceding tables (p. 557 and 569), are adapted to the specific gravities here marked, and which have been adopted from the best authorities.

The malleable metals are supposed to be hammered as much as they usually are, when they are wrought into shape.

The different kinds of stone have been selected amongst those which are in most common use; and the specific gravities marked above are the averages of different specimens. The gage points marked stone, are adapted to a specific gravity of 2·48, or 155 lbs. to a cubic foot.

The specific gravity of wood is taken at an intermediate state between unseasoned wood, and extremely dry wood, being the state in which it is fit for carpenters' use; but if the wood is cut into thin planks and dried sufficiently for joiners and cabinet makers, it will become lighter. Different specimens of the same tree, will differ very considerably in specific gravity.

The weight of a cubic foot of such substances as are in most common use, is given in pounds, to facilitate calculations; it may be obtained in other cases by multiplying the specific gravity of the substance by $62\frac{1}{2}$ lbs., which is the weight of a cubic foot of rain water; or dividing the specific gravity by ·016 (the reciprocal of 62·5) will give the same result.

There is nothing peculiar in the use of this new sliding rule, for ordinary purposes, which can require any specific directions. Those precepts which are marked C and D, are to be performed with the two upper lines of the new rule which are marked C and D. The slide may be inverted, when necessary, for the precepts marked ꓭ and D, but which will be C and ∀ on the new rule. When the precepts are marked slide inverted A and Ɔ, the two lower lines A and ꓭ of the new rule are to be used without inverting the slider, because it is made inverted. For cases which are marked A and B simply, the slide must be inverted, and the two lower lines employed as direct lines, but they are not so well adapted for this as for the other cases.

To enable all the four lines of the new rule to be used in concert, in the same manner as those of the Soho rule, the 10 in the middle of the broken inverted line ɡ, at the bottom of the rule, is placed exactly opposite to the 10 at the middle of the line C, at the top of the rule. This is not correctly represented in the sketch, but it may be easily imagined, and it enables the rule to perform some compound operations which cannot be done on other sliding rules: for instance,

To multiply two numbers together, and multiply their product by the square root of a third number. For instance, multiply 8 by 4, and multiply their product by the square root of 16.

Thus, the two factors are 8 and 4; and the square of the multiplier is 16, the root of which is 4 for the real multiplier. The resulting product is $(8 \times 4 = 32 \times 4 =)$ 128.

The author's Sliding Rule, all four lines.	C	Square of multiplier.	*Example.*	C	16 square of mult.
	D	Resulting product.		D	128 result.
	A	One of the factors.		A	4 one factor
	ɡ	Other factor.		ɡ	8 other factor.

Note. If the intermediate product of the multiplication is required, it may be found upon either of the lines A or ɡ opposite to 1 upon the other line. The series of numbers on the line C is twice repeated, and a different resulting product will be found opposite to each of the repetitions of the number which represents the square of the multiplier; the rule does not point out which of these results is the true one, but it can be known by other means.

Use of the line of one and a half radius on the Author's New Sliding Rule.

The line of divisions marked ȝ, at the back of the slider of the author's new sliding rule, is a logarithmic scale like the others; but the length of its radius, or the distance from 1 to 10, is only two-thirds of the length of the radius, or the distance from 1 to 10, upon the line of single radius, ɡ.

The use of this additional line is to enable us to raise the 1·5th power of any number; that is a fractional power which is less than the square of the number; being equal to the square root of the cube of the number.

This fractional power is sometimes called the power of three halves: it may be raised by multiplying the logarithm of the number by 1·5, instead of by 2, or by 3, as must be done if it were required to raise the square or cube of the number.

Example. To raise the three-half power of 4. The cube of 4 is 64, the square root of which is 8, for the 1·5th power of 4, as required. Or by logarithms; the log. of 4 is 0·602 × 1·5 = 0·903, which is the log. of 8.

To use the line ȝ, the slider must be taken out of its groove and turned upside down, and then replaced in the groove, with the line ȝ applied against the lower line ɡ of the rule; the numbers of both lines will then count the same way, because both are inverted lines; and if the slider is set so that 1 (or 31·623) upon the line ȝ corresponds with 1 (or 10) upon the line B, then any number being chosen upon the lower line ɡ, the 1·5th power of that number will be found opposite to it on the line ȝ, thus,

The author's Sliding Rule.	ȝ 1 or 31·6	1·5th power of numb.	or	Number.	*Ex.*	ȝ 31·6 8 power.	or	64 num.
	ɡ 1 or 10	Number.		1·5th root.		ɡ 10 4 numb.		16 root.

And conversely the lines ȝ and ɡ, serve to extract the 1·5 root of a number; which is the same thing as the square of the cube root of the number.

Example. To extract the three half root of 64. The cube root of 64 is 4; the square of which is 16, for the 1·5th root of 64, as required. Or the logarithm of 64 is 1·806 ÷ 1·5 = 1·204, which is the log. of 16.

The principal use of the line ȝ is for facilitating calculations relative to the discharge of water through apertures, such as sluices, or in cascades over weirs, or mill-dams, thus.

To calculate the quantity of water which will flow in a cascade over the edge of a weir, or through an aperture or notch in the edge of a board, such notch or aperture being open at top. Having given the depth in inches, from the level surface of the water in the reservoir, down to the edge over which the cascade of water flows; to find the quantity of water in cubic feet, which will be discharged per minute, over one foot wide of such cascade.

RULE. Multiply the square root of the cube of the depth in inches, by 5, and the product will be the number of cubic feet which will be discharged per minute over every foot in width; and therefore by multiplying the product by the width of the cascade in feet, the number of cubic feet discharged per minute will be obtained.

Example. What quantity of water will cascade over a weir 20 feet wide, when the level of the water in the reservoir is 4 inches above the edge of the weir. 4 inc. deep cubed = 64, the square root of which is 8 × 5 = 40 cubic feet per minute will be discharged over each foot wide; and × 20 feet wide = 800 cubic feet will be discharged per minute.

The author's new Sliding Rule.	Ǝ	40 cubic ft. per min. over 1 ft. wide.	*Example.*	Ǝ	40	158 cub. ft.
	ꓭ	4 inches deep below level surface.		ꓭ	4	10 inches.

The rule being thus set, the two lines form a table; the numbers on the line Ǝ showing how many cubic feet which will be discharged per minute over a cascade one foot wide, at that depth in inches, which is denoted by the corresponding numbers on the line ꓭ.

Note. This table only extends from 3 inches deep, to 30 inches deep; it will comprise most cases in practice, but to make the rule serve as a table from ·3 tenths to 3 inches deep, it must be set as follows.

The author's new Sliding Rule.	Ǝ	5 cubic ft. per min. over 1 ft. wide.	*Exam.*	Ǝ	1	1·77 cub. ft.
	ꓭ	1 inch deep below level surface.		ꓭ	5	·5 of an inc.

The rule when thus set will also serve as a table from 30 inches deep to 300 inches deep, if the value of the divisions is estimated as follows.

The author's new Sliding Rule.	Ǝ	5000 cubic ft. per min. over 1 ft. wide.	*Exam.*	Ǝ	5000	1560 cub ft.
	ꓭ	100 inches deep below level surface.		ꓭ	100	46 inches.

The following table will point out the proper value for the divisions in these computations.

Depth inches.	Cubic feet per minute.	Depth inches.	Cubic feet per minute.	Depth inches.	Cubic feet per minute.	Depth inches.	Cubic feet per minute.	Depth inches.	Cubic feet per minute.
$\frac{1}{4}$	625	4	40	13	234·36	22	515·94	40	1264·9
$\frac{1}{2}$	1·767	5	55·9	14	261·91	23	551·52	50	1768
$\frac{3}{4}$	3·248	6	73·48	15	290·47	24	587·87	60	2323·8
1	5	7	92·60	16	320	25	625	70	2928·3
$1\frac{1}{2}$	9·185	8	113·14	17	350·46	26	664	80	3577·7
2	14·14	9	135·	18	381·83	27	701·57	90	4269
$2\frac{1}{2}$	19·76	10	158·11	19	414·1	28	740·8	100	5000
3	25·98	11	182·41	20	447·22	29	780·85	200	14142
$3\frac{1}{2}$	32·74	12	207·85	21	481·17	30	821·58	300	25980

Note. The above rule will give results which correspond very nearly with the best experiments on the discharge of water through apertures. The quantity discharged is assumed to be ·65 of that which might be expected, according to the theory of bodies falling freely in vacuo (see p. 23 and 272); because the flowing water cannot acquire its full velocity all at once. Whilst the water is actually passing through the aperture, it usually moves with about ·65 of the velocity that a body would acquire by falling through a height equal to the depth of the water; but the motion being accelerated, the stream acquires very nearly that velocity after it has quitted the aperture.

The method of calculation best adapted for the purposes of engineers being now explained, we may proceed, in the next chapter, to state a number of specific rules by which the dimensions of every important part of a steam-engine of any size, may be calculated with great facility, so as to obtain the same proportions as those which were given by Mr. Watt to his engines.